Macroeconometrics: Final Exam

PSE - Spring 2015

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Answer all questions. The due date is the 23:59 on May 8, 2015. Email me if something is not clear. All answers must be type in. Good luck.

Question 1

Consider an ARMA(1,1) model:

$$y_t = \phi y_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1}, \qquad \varepsilon_t \sim iidN(0, \sigma^2).$$

1. What are the stationarity conditions for this process?

2. What are the invertibility conditions for this process?

3. Assume the model is stationary. Express the process in the Wold form and specify Wold form coefficients in terms of ϕ and θ.

4. Consider the case in which $\phi = -\theta$.

   (a) How does the ACF look like?

   (b) What can you say about this process?

Question 2

Consider the unobserved component model with AR(2,1) structure for the transitory component, c_t:

$$\begin{aligned} y_t &= \tau_t + c_t, \\ \tau_t &= \mu + \tau_{t-1} + \eta_t, \qquad \eta_t \sim N(0, \sigma_\eta^2) \\ c_t &= \phi_1 c_{t-1} + \phi_2 c_{t-2} + \varepsilon_t + \theta \varepsilon_{t-1}, \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \\ & cov(\eta_t, \varepsilon_t) = \sigma_{\eta\varepsilon} \end{aligned}$$

1. Under what conditions $c_t \sim I(0)$?

2. Show that without making any assumption about the correlation between permanent and transitory component, $\sigma_{\eta\varepsilon}$, this model is not identified.

3. Show that under the assumption $\sigma_{\eta\varepsilon} = 0$, the model is identified.

4. What does this assumption, i.e. $\sigma_{\eta\varepsilon} = 0$, mean?

5. Cast the model in a state space form and write how it can be estimated using maximum likelihood. That is, describe how you would compute and maximize the log-likelihood function..

Question 3

Consider the following dynamic simultaneous equations model

$$
\begin{aligned}
y_{1t} &= \beta_{12} y_{2t} + \gamma_{11} y_{1t-1} + \gamma_{12} y_{2t-1} + \varepsilon_{1t} \\
y_{2t} &= \beta_{21} y_{1t} + \gamma_{21} y_{1t-1} + \gamma_{22} y_{2t-1} + \varepsilon_{2t}
\end{aligned}
$$

1. Write down this model as a structural VAR model, *i.e.* in a matrix form of

$$\mathbf{B}\mathbf{y_t} = \mathbf{\Gamma}\mathbf{y_{t-1}} + \varepsilon_\mathbf{t},$$

   where $\mathbf{y_t} = (y_{1t}, y_{2t})'$, $\varepsilon_\mathbf{t} = (\varepsilon_{1t}, \varepsilon_{2t})'$, and $\mathbf{B}$ and $\mathbf{\Gamma}$ are matrices of parameters. It is assumed that error term satisfies $\varepsilon_\mathbf{t} \sim iid(0, \mathbf{\Sigma})$, and $\mathbf{\Sigma}$ is diagonal with σ_i^2 $(i = 1, 2)$.

2. Determine the reduced form of the VAR model

$$\mathbf{y_t} = \mathbf{A}\mathbf{y_{t-1}} + \mathbf{e_t}$$

   and use it to show that the parameters of the structural VAR(1) are not identified without further restrictions.

3. What kind of restrictions are usually put on the parameters of the structural VAR(1) in order to achieve identification?

4. Suppose $\mathbf{B}^{-1}$ is lower triangular matrix. What does this imply for the structural model? How can this be used to identify the VAR model?

5. Suppose you want check if y_{1t} Granger-causes y_{2t}. How can you do it using the reduced-form VAR(1)? If you find that y_{1t} Granager causes y_{2t}, what does this imply for the parameters of our structural VAR model?

Question 4

Consider a Purchasing Power Parity where the nominal exchange rate, e_t, can be expressed as

$$e_t = p_t - p_t^* + v_t, \qquad v_t \sim iid(0, \sigma_v),$$

where both the price level in the foreign country has a unit root,

$$p_t^* = p_{t-1}^* + u_{1t},$$

and the domestic price has a unit root,

$$p_t = p_{t-1} + u_{2t},$$

and $u_{it} \sim I(0)$ for $i = 1, 2$.

1. If PPP holds, we say that the e_t, p_t and p_t^* are cointegrated, that is, there exists a linear combination of e_t, p_t and p_t^* that is stationary. In particular, PPP implies that the real exchange rate, z_t,

$$z_t = e_t - p_t + p_t^*,$$

is stationary. How would you test if the real exchange rate, z_t, is indeed stationary?