Time Series Econometrics
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Homework 1: Solutions

Homework 1: Question 1
Consider an ARMA(1,2) model:

$$y_t = \phi y_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}, \qquad \varepsilon_t \sim iidN(0, \sigma^2).$$

(a) What are the stationarity conditions for this process?

(b) What are the invertibility conditions for this process?

(c) Express the process in the Wold form and specify Wold form coefficients in terms of ϕ and θ_is.

Answer:

(a) ARMA(1,2) process is stationary if and only if $|\phi| < 1$.

(b) ARMA(1,2) is invertible when roots of characteristic equation $\theta(z) = 1 + \theta_1 z + \theta_2 z^2 = 0$ are outside the unit circle, i.e.

$$|z_{1,2}| = \left| \frac{-\theta_1 \pm \sqrt{\theta_1^2 - 4\theta_2}}{2\theta_2} \right| > 1$$

One can show that it corresponds to the following restrictions on θ_1 and θ_2:

$$\theta_1 + \theta_2 > -1, \qquad \theta_1 - \theta_2 < 1, \qquad |\theta_2| < 1.$$

(c) The Wold form in terms of ϕ and θ_is, for $|\theta| < 1$, can be written as

$$y_t = \sum_{i=0}^{\infty} \psi_i \varepsilon_{t-i}$$

where

$$\begin{aligned} \psi_0 &= 1 \\ \psi_1 &= \phi + \theta_1 \\ \psi_i &= \phi^i + \theta_1 \phi^{i-1} + \theta_2 \phi^{i-2}, \qquad \forall i \geq 2. \end{aligned}$$

Homework 1: Question 2
Stata Exercise