

Homework 2

Due date: Wednesday, 8 May, 13:45

1. Question: state-space models

(a) Give the state-space representation of the following models (remember to specify matrices Q and R):

i. Unobserved component model with a stochastic drift

$$\begin{aligned}y_t &= \tau_t + c_t, \\ \tau_t &= g_{t-1} + \tau_{t-1} + v_t, \quad v_t \sim iid N(0, \sigma_v^2) \\ g_t &= g_{t-1} + w_t, \quad w_t \sim iid N(0, \sigma_w^2) \\ c_t &= \phi_1 c_{t-1} + \phi_2 c_{t-2} + e_t, \quad e_t \sim iid N(0, \sigma_e^2).\end{aligned}$$

Note that in this model, the stochastic trend, τ_t , has a drift term (g_t) modeled as a random walk.

ii. Time-varying parameter model

$$\begin{aligned}y_t &= x_{1,t}\beta_{1,t} + x_{2,t}\beta_{2,t} + u_t, \quad u_t \sim iid N(0, \sigma^2), \\ \beta_{i,t} &= \beta_{i,t-1} + v_{i,t}, \quad v_{i,t} \sim iid N(0, \sigma_i^2).\end{aligned}$$

(b) Briefly discuss how you would estimate a model given in a state space form using maximum likelihood.