

## Homework 2

Due date: Wednesday, 24 May 2023, 13:30

### 1. Question 1

- (a) Give the state-space representation of the following models:
- Unobserved component model with a stochastic drift

$$\begin{aligned}y_t &= \tau_t + c_t, \\ \tau_t &= g_{t-1} + \tau_{t-1} + v_t, \quad v_t \sim iid N(0, \sigma_v^2) \\ g_t &= g_{t-1} + w_t, \quad w_t \sim iid N(0, \sigma_w^2) \\ c_t &= \phi_1 c_{t-1} + \phi_2 c_{t-2} + e_t, \quad e_t \sim iid N(0, \sigma_e^2).\end{aligned}$$

Note that in this model, the stochastic trend,  $\tau_t$ , has a drift term ( $g_t$ ) modeled as a random walk.

- Time-varying parameter model

$$\begin{aligned}y_t &= x_{1,t}\beta_{1,t} + x_{2,t}\beta_{2,t} + u_t, \quad u_t \sim iid N(0, \sigma_u^2), \\ \beta_{i,t} &= \beta_{i,t-1} + v_{i,t}, \quad v_{i,t} \sim iid N(0, \sigma_i^2).\end{aligned}$$

- (b) Briefly discuss how you would estimate a model given in a state space form using maximum likelihood.

### 2. Question 2: Sims (1980)

Using the sample code from the website estimate a  $3 \times 3$  VAR model for growth rate of real GDP per capita, federal funds rate, and CPI in the model with short-run restrictions.

- For the data, for the bonus points go to the FRED Economic Data website and download the most up-to-date data. You will have to convert the federal funds rate and CPI data to quarterly data. Explain how you choose to convert your data to the quarterly frequency. Or use the data from the class website and load them `import excel freddata.xlsx, sheet("FREDStata") firstrow`. Note that the GDP data in the file are already in logs.
- Set the time/date indicator with `tsset`. You can either do as in homework 1 or (if you use the file from the website) use series `index` that is loaded with the dataset.
- Generate the CPI inflation rate and the rate of growth of real GDP per capita.
- Write down the VAR model you want to estimate. What do you assume about the contemporaneous effects of the variables in that model? What are your short run restrictions?

- (e) Estimate a VAR model with short-run restrictions as in Sims (1980) and produce IRF's. Comment on the results.
- (f) Change the order of the variables in the VAR vector. What are the new short run restrictions you are imposing? How are your results changing?