Lecture: Impulse Response Functions and Local Projections

222061-1617: Time Series Econometrics

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[1]Based on notes by A. Zervou

Outline

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1. Intro/Setting
2. Identification
3. Impulse Response Functions

Questions

What are the causal effects of changes in:

- government spending, taxes
- monetary policy
- technology, preferences

on output, consumption, investment, prices, interest rates?

In order to find causal effects, we encounter identification, which turns correlations into causation.

Steps

- Identify shocks.
- Uncover causal effects.
- Study impulse response functions.

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IRF's

- You now have identified the shock(s), i.e., VAR identification or exogenous series shocks.
- If you use VAR for identification and is well specified, use VAR for IRF's.
- BUT: If you do not use VAR for identification or if your VAR it is not well specified, then what?
- Use Jorda (2005) local projection method for estimating impulse responses.

Remember: Impulse Response Functions VAR

- VAR IRFs: give us the effect of the shocks on endogenous variables.
- E.g., we want to know: $\frac{\partial y_{2,t}}{\partial \varepsilon_{1,t}}, \frac{\partial y_{2,t+1}}{\partial \varepsilon_{1,t}}, \frac{\partial y_{2,t+2}}{\partial \varepsilon_{1,t}}, \frac{\partial y_{2,t+3}}{\partial \varepsilon_{1,t}}$ etc.
- We employ stationarity, so $\frac{\partial y_{2,t+1}}{\partial \varepsilon_{1,t}} = \frac{\partial y_{2,t}}{\partial \varepsilon_{1,t-1}}, \frac{\partial y_{2,t+2}}{\partial \varepsilon_{1,t}} = \frac{\partial y_{2,t}}{\partial \varepsilon_{1,t-2}}, \frac{\partial y_{2,t+3}}{\partial \varepsilon_{1,t}} = \frac{\partial y_{2,t}}{\partial \varepsilon_{1,t-3}}$, etc.
- $\frac{\partial y_{2,t}}{\partial \varepsilon_{1,t-1}}, \frac{\partial y_{2,t}}{\partial \varepsilon_{1,t-2}}, \frac{\partial y_{2,t}}{\partial \varepsilon_{1,t-3}}$... can be estimated by the structural MA form of the VAR.

Remember: Impulse Response Functions: Example

2x2 example:

- structural MA form:

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \begin{bmatrix} \theta_{11,0} & \theta_{12,0} \\ \theta_{21,0} & \theta_{22,0} \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} + \begin{bmatrix} \theta_{11,1} & \theta_{12,1} \\ \theta_{21,1} & \theta_{22,1} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1} \\ \varepsilon_{2,t-1} \end{bmatrix} +$$

$$+ \begin{bmatrix} \theta_{11,2} & \theta_{12,2} \\ \theta_{21,2} & \theta_{22,2} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-2} \\ \varepsilon_{2,t-2} \end{bmatrix} + \begin{bmatrix} \theta_{11,3} & \theta_{12,3} \\ \theta_{21,3} & \theta_{22,3} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-3} \\ \varepsilon_{2,t-3} \end{bmatrix} + \ldots$$

- Then, $\theta_{21,s} = \frac{\partial y_{2,t}}{\partial \varepsilon_{1,t-s}} \left(= \frac{\partial y_{2,t+s}}{\partial \varepsilon_{1,t}} \right)$
- E.g., $\theta_{21,3} = \frac{\partial y_{2,t}}{\partial \varepsilon_{1,t-3}} \left(= \frac{\partial y_{2,t+3}}{\partial \varepsilon_{1,t}} \right)$
- E.g., $\theta_{11,2} = \frac{\partial y_{1,t}}{\partial \varepsilon_{1,t-2}} \left(= \frac{\partial y_{1,t+2}}{\partial \varepsilon_{1,t}} \right)$
- E.g., $\theta_{22,1} = \frac{\partial y_{2,t}}{\partial \varepsilon_{2,t-1}} \left(= \frac{\partial y_{2,t+1}}{\partial \varepsilon_{2,t}} \right)$
- VAR: backward looking IRFs, given stationarity.

Remember: VAR Impulse Response Functions

- Use MA form of VAR
- Iterate equation h periods ahead to get the h^{th} period IRFs
- E.g., $\frac{\partial y_{1,t+h}}{\partial \varepsilon_{1t}} = \theta_{11,h} = \left(\frac{\partial y_{1,t}}{\partial \varepsilon_{1t-h}}\right)$
- Thus, VAR IRFs are iterated IRFs.

Remember: VAR Impulse Response Functions

- Remember your notes from a 2x2 VAR:
- Structural IRF's is the reduced form IRFs times inverse of contemporaneous effects matrix:

$$\begin{bmatrix} \theta_{11,h} & \theta_{12,h} \\ \theta_{21,h} & \theta_{22,h} \end{bmatrix} = \begin{bmatrix} \psi_{11,h} & \psi_{12,h} \\ \psi_{21,h} & \psi_{22,h} \end{bmatrix} \frac{1}{1 - b_{12} b_{21}} \begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix}$$

- E.g., $\frac{\partial y_{1,t+h}}{\partial \varepsilon_{1t}} = \theta_{11,h} = \frac{1}{1-b_{12}b_{0,21}} \left(\psi_{11,h} + \ b_{0,21} \psi_{12,h} \right)$
- Thus, VAR structural IRF's are nonlinear functions of reduced form VAR estimated parameters.

VAR Impulse Response Functions

VAR irfs:

- Iterated IRFs, not direct
- If model does not capture well the data generating process, there are specification errors, compounding with horizon
- Structural IRF's are nonlinear functions of reduced form VAR estimated parameters
- Produced through VAR identification.

Local Projections

Local Projections IRF's

- Local projections give direct IRFs vs. iterated VAR IRFs (the discussion is analogous to iterated vs. direct forecasting we did earlier)
- We do not need to impose restrictions on variables relationships
- Direct estimation: they do not result from non-linear relationships of estimated parameters
- Does not require VAR identification.

IRF's

- Let $\varepsilon_{1,t}$ shock, identified in any method discussed earlier
- In general, an IRF is

$$\mathrm{E}[Y_{i,t+h}|\varepsilon_{1,t} = 1; controls] - \mathrm{E}[Y_{i,t+h}|\varepsilon_{1,t} = 0; controls],$$

  where E is expectations operator.
- IRF gives us the effect of a shock $\varepsilon_{1,t}$ occurring at time t, on the endogenous variable Y at time $t+h$
- VARs is one way to produce such kind of IRFs.

Local Projections IRF's

Local projections IRF's:

- Estimate a single regression of the identified shock $\varepsilon_{1,t}$ on Y_{t+h}, for each horizon h
- E.g., for horizon h:

$$Y_{i,t+h} = \theta_{i1,h}\, \varepsilon_{1,t} + controls + \xi_{i,t+h}$$

- $\theta_{i1,h}$ is the local projection IRF estimate at horizon h (from the model where Y_i and ε_1 have h lags difference; please remind yourself of the forecasting lectures notation).

Local Projections IRF's: Example

Estimate a single regression of the identified shock $\varepsilon_{1,t}$ on Y_{t+h}, for each horizon h

- horizon 0:
$$Y_{i,t} = \theta_{i1,0}\, \varepsilon_{1,t} + controls + \xi_{i,t}$$
- horizon 1:
$$Y_{i,t+1} = \theta_{i1,1}\, \varepsilon_{1,t} + controls + \xi_{i,t+1}$$
- horizon 2:
$$Y_{i,t+2} = \theta_{i1,2}\, \varepsilon_{1,t} + controls + \xi_{i,t+2}$$
- ....
- horizon h:
$$Y_{i,t+h} = \theta_{i1,h}\, \varepsilon_{1,t} + controls + \xi_{i,t+h}$$
- $\theta_{i1,0}, \theta_{i1,1}, \theta_{i1,2} \ldots \theta_{i1,h}$ gives the IRF's estimates of the shock.

Local Projections IRF's: Example

Estimate a single regression of the identified shock $\varepsilon_{1,t}$ on Y_{t+h}, for each horizon h.

- horizon 0:

$$Y_{i,t} = \theta_{i1,0}\ \varepsilon_{1,t} + controls + \xi_{i,t}$$

- horizon 1:

$$Y_{i,t+1} = \theta_{i1,1}\ \varepsilon_{1,t} + controls + \xi_{i,t+1}$$

- horizon 2:

$$Y_{i,t+2} = \theta_{i1,2}\ \varepsilon_{1,t} + controls + \xi_{i,t+2}$$

- ....
- horizon h:

$$Y_{i,t+h} = \theta_{i1,h}\ \varepsilon_{1,t} + controls + \xi_{i,t+h}$$

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Local Projections IRF's: Example

Estimate a single regression of the identified shock $\varepsilon_{1,t}$ on Y_{t+h}, for each horizon h.

- horizon 0:

$$Y_{i,t} = \theta_{i1,0}\ \varepsilon_{1,t} + controls + \xi_{i,t}$$

- horizon 1:

$$Y_{i,t+1} = \theta_{i1,1}\ \varepsilon_{1,t} + controls + \xi_{i,t+1}$$

- horizon 2:

$$Y_{i,t+2} = \theta_{i1,2}\ \varepsilon_{1,t} + controls + \xi_{i,t+2}$$

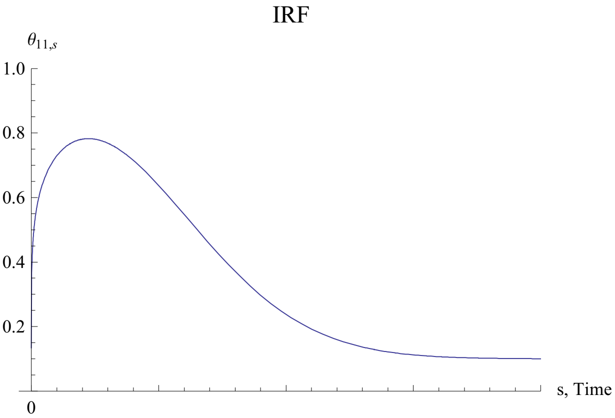
- ....
- horizon h:

$$Y_{i,t+h} = \theta_{i1,h}\ \varepsilon_{1,t} + controls + \xi_{i,t+h}$$

- $\theta_{i1,0}, \theta_{i1,1}, \theta_{i1,2} \ldots \theta_{i1,h}$ gives the IRF's estimates of the shock.

Impulse Response Functions

$\theta_{i1,h}$



Local Projections IRF's: Controls

$$Y_{i,t+h} = \theta_{i1,h}\, \varepsilon_{1,t} + controls + \xi_{i,t+h}$$

Controls:

- Note: Each horizon is a different equation and can use different controls: *flexibility*
- Control variables need not include the other $Y_{i,t}$s (if $\varepsilon_{1,t}$ exogenous to those $Y_{i,t}$s)
- Common controls: deterministic part (constant, time trends), lags of the Y_is, and lags of other variables
- The specification can be chosen using information criteria.

Local Projections IRF's-autocorrelation

$$Y_{i,t+h} = \theta_{i1,h}\, \varepsilon_{1,t} + controls + \xi_{i,t+h}$$

Autocorrelation:

- Except for horizon $h = 0$, error terms serially correlated (i.e., $\xi_{i,t}$ correlates with $\xi_{i,t+1}$, $\xi_{i,t+2}$, *etc.*)
- That's because error term at h is moving average of forecast errors from t to $t + h$
- Thus use corrected for autocorrelation standard errors, Newey-West standard errors, with h lags specification.

Local Projections IRF's vs. VAR IRFs

Local Projections IRFs:

- Direct IRF's, i.e., directly estimate the effect of $\varepsilon_{1,t}$ on $Y_{i,t+h}$ (vs. iterated through VARs)
- Could use different control variables for each horizon
- Fewer imposed restrictions, so less precisely estimated IRFs
- MA error terms, so use HAC standard errors, e.g., Newey-West (1987) correction
- Single-equation estimations and thus flexible method, can incorporate:
    - state dependent effects i.e. different effects during recessions or expansions, positive changes different than negative, etc
    - IV estimations
    - multiple instruments
    - everything that you can do in single-equation environemnt

Local Projections: Examples of Flexibility

1) Local Projections IRF's with IV

- You want to estimate the effect of Y_1 on Y_i, i.e., estimate:

$$Y_{i,t+h} = \theta_{i1,h}\, Y_{1,t} + controls + \xi_{i,t+h}$$

- Usual issue: Y_1 is correlated with $\xi_{i,t+h}$
- Use external instrument Z_t, as instrument for $Y_{1,t}$
- E.g., Y_i is real output, $Y_{1,t}$ is federal funds rate, then we use narrative Romer and Romer's shock series as instrument

2) Local Projections, IVs and Asymmetries Example

- E.g., Kilian and Vigfusson's (2011) testing for asymmetries.
- Y is linear function of X, where X takes on both negative and positive values:
$$Y_t = \beta X_t + \varepsilon_t$$
- But you instead estimate:
$$Y_t = \beta' X_t^+ + \varepsilon_t$$
  where X_t^+ are only the positive values of X.
- Suppose you find that β' is bigger (in absolute value) than β
- Do you conclude that there are asymmetries, with positive values X having bigger effects than negative values?
- No: this specification leads to bias because you are implicitly setting all negative values of X to zero.

Asymmetries Example

Quantitative Economics 2 (2011) Are responses of the U.S. economy asymmetric? 425

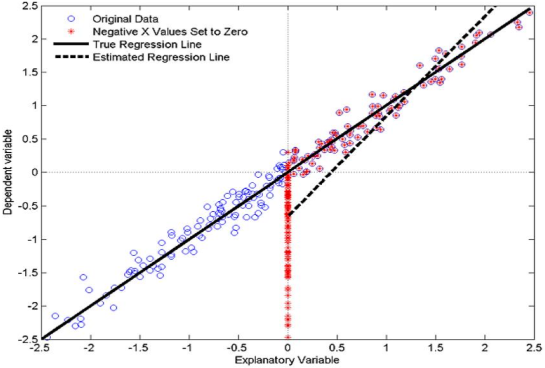


FIGURE 1. The effect of censoring negative values of the explanatory variable.

Local Projections, IVs and Asymmetries Example

- The procedure that truncates on the X variable produces slope coefficients biased upward in magnitude
- Incorrectly conclude that positive Xs have greater impact than negative Xs, even when the true relationship is linear
- They use an example from the oil literature that had found that oil price increases had bigger effects than oil price decreases
- To guard against this faulty inference, one should always make sure that the model nests the linear case when one is testing for asymmetries
- You should estimate:

$$Y_t = \beta X_t + \beta' X_t^+ + \varepsilon_t$$

and test whether $\beta' = 0$.

Local Projections, IVs and Asymmetries Example

- Often narrative methods focus only on one side
- IV can help
- E.g.,

$$Y_t = \alpha + \beta X_t + \varepsilon_t,$$

  with $E(X_t\ \varepsilon_t) \neq 0$.
- Let

$$X_t = \gamma + \theta Z_t + u_t,$$

  with Z_t independent of ε_t
- Suppose you have only positive values of Z, Z^+
- Given that Z exogenous, functions of Z is exogenous, including Z^+
- So use Z^+ as instrument of X_t
- E.g., Z^+ is a narrative of tax increases during fiscal consolidations and X might be tax revenue or the deficit
- Then use local projections for estimating IRF's.

Conclusions

- New identification methods (e.g., narrative, high frequency) offer alternative/additional ways to identify macroeconomic shocks and examine their effects on endogenous macro variables
- Those new identified series can be used as the actual shocks, or as noisy measures of the actual shocks (IV-approach)
- Jorda local projections method produces IRFs if we choose to use one-equation estimation approach

Conclusions Local projections vs. VAR IRFs

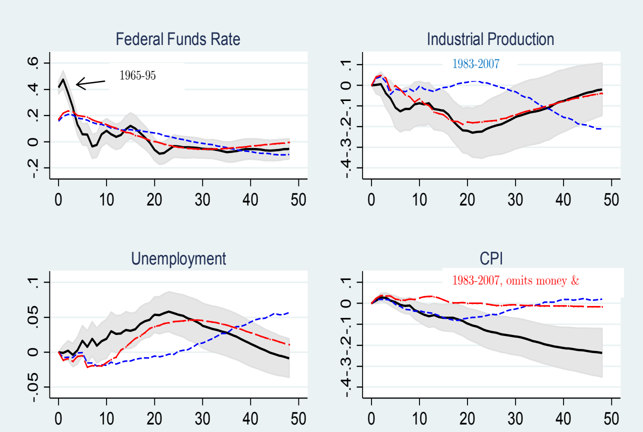
- Local projections estimation is simple and able to cope with additional issues like state-dependence; VARs are usually large systems, hard to estimate and hard to add additional complications
- Local projections IRFs are direct IRFs instead of the VAR iterated IRFs
- For Local projections IRFs each horizon can be estimated using different controls
- Local projections IRFs do not depend on the validity of imposed restrictions (but of course, do depend on the quality of the identified shock or instrument)
- Given the construction of local projections IRFs, they are less precise and might look more choppy than VAR IRF's, which are in general smoother (remember how those are constructed)

Conclusions Local projections vs. VAR IRFs

- See for example the VAR and Jorda monetary policy shock IRFs below (look at black lines only, effects of monetary policy shocks on Industrial Production, CPI, Federal Funds Rate and Unemployment)
- Notice that for the same variable, the IRFs of Jorda look more unstable than the VAR IRFs

1) VAR with short-run restrictions

Figure 3.1. Christiano, Eichenbaum, and Evans (1999) Identification
1965m1-1995m6 full specification: solid black lines; 1983m1-2007m12 full specification: short dashed blue lines; 1983m1-2007m12, omits money and reserves: long dashed red lines)



2) Narrative shocks used as Instrument, Jorda

B. Jordà Local Projection Method, 1969m3–1996m12 Recursiveness assumption: solid black lines; No recursiveness assumption: dashed green lines; No recursiveness assumption, FAVAR controls: dashed purple lines.