# Lecture 10: Money in OLG Model

Advanced Macroeconomics
University of Warsaw

Jacek Suda

June 7, 2017

# OLG model with Money

- Time is discrete and continues forever: $t = 1, 2, 3 \ldots$
- In each period, $L_t$ two-period-lived agents are born.
- In period 1 there are $L_0$ *old* agents who live for one period.
- Population evolves according to $L_t = (1+n)^t L_0$, with $n > -1$.
  (if $n = 0$ no population growth)

# Agents

- An agent born at date $t$ has preferences represented by
$$u(c_t^t, c_{t+1}^t)$$
  - $c_t^t$ denotes the consumption at date $t$ of an agent born in period $t$ (*young* agent) .
  - $c_{t+1}^t$ denotes the consumption at date $t+1$ of an agent born in period $t$ (*old* agent).
- $u(\cdot,\cdot)$ is utility function that is strictly increasing in both arguments, strictly concave, twice continuously differentiable, and for
$$v(c_y, c_o) \equiv \frac{\frac{\partial u(c_y,c_o)}{\partial c_y}}{\frac{\partial u(c_y,c_o)}{\partial c_o}}.$$
we have
$$\lim_{c_y \to 0} v(c_y, c_o) = \infty, \qquad \text{for any } c_o > 0,$$
$$\lim_{c_o \to 0} v(c_y, c_o) = 0, \qquad \text{for any } c_y > 0,$$
  - agents want to consume positive amounts in both periods of life.

# Money

- An agent receives $y$ units of perishable consumption good in the first period (when young), and nothing in the second period (when old),
    - agent's endowment profile is $e = (e_t^t, e_t^{t+1}) = (y, 0)$.
- $L_0$ initial old agents are collectively endowed with $M_0$ units of fiat money.
- Money is perfectly divisible, intrinsically useless, and can not be privately produced.
- Money are injected/withdrawn through lump-sum transfers to old agents in each period, $\tau_t$.

# Money

- Total money supply in period $t$ equals $M_t$ and the government budget constraint is
$$p_t(M_t - M_{t-1}) = L_{t-1}\tau_t,$$
  - $p_t$ denotes the price of money in terms of the period $t$ consumption good,
  - $p_t = 0$ implies the case where money has no value.
- Money supply grows at a constant rate
$$M_t = zM_{t-1} \qquad z \geq 0.$$
- The government budget constraint becomes
$$p_t M_t(1 - \frac{1}{z}) = L_{t-1}\tau_t.$$

# Social Planner

- Consider a social planner that can re-distribute young agents' endowments across the population.
- The resource constraint is given by

$$L_t c_t^t + L_{t-1} c_{t-1}^t \leq L_t y, \qquad t = 1, 2, \ldots$$

  - in each period, total consumption of the young ($L_t c_t^t$) plus total consumption of the old ($L_{t-1} c_{t-1}^t$) can not exceed the total endowment ($L_t y$).
- Assume planner chooses a stationary allocations, i.e. $(c_t^t, c_t^{t+1}) = (c_y, c_o)$, for all $t$.
- The resource constraint becomes

$$c_y + \frac{c_o}{1+n} \leq y$$

and we call any allocation $(c_y, c_o)$ that satisfies this equation *feasible*.

# Definition: Pareto optimal allocation

**Definition**

A *Pareto optimal allocation*, chosen from the class of stationary allocations, $(c_y, c_o)$, is feasible and satisfies the property that there exists no other feasible stationary allocation $(\hat{c}_y, \hat{c}_o)$ such that

$$u(\hat{c}_y, \hat{c}_o) \geq u(c_y, c_o),$$
$$\hat{c}_o \geq c_o$$

with at least one of these two inequalities begin a strong inequality.

# Stationary social planner optimal allocation

- The stationary solution to the planner problem solves

$$\max_{c_y, c_o} u(c_y, c_o)$$

subject to

$$c_y + \frac{c_o}{1+n} = y \ .$$

- The interior solution $(c_y^*, c_o^*)$ satisfies

$$-u_1(c_y^*, c_o^*) + (1+n)u_2(c_y^*, c_o^*) = 0 \ .$$

# Young Consumer's Problem

- An agent born at date $t$ solves the problem

$$\max_{c_t^t, c_{t+1}^t, m_t} u(c_t^t, c_{t+1}^t)$$

subject to budget constraints

$$c_t^t + p_t m_t \leq y,$$

$$c_{t+1}^t \leq p_{t+1} m_t + \tau_{t+1},$$

  - $m_t$ is a nominal quantity of money acquired by *young* agent,
  - $\tau_{t+1}$ is money transfers from government to old agents at date $t+1$,

- Agents choose money holdings, $m_t$, and consumption treating sequence of prices $p_t, p_{t+1}$ and money transfers $\tau_t$ as given.

# Definition: Competitive Equilibrium

**Definition**

A *competitive equilibrium* is a sequence of consumption allocations $\{(c_t^t, c_{t+1}^t)\}_{t=1}^{\infty}$, a sequence of individual money demands $\{m_t\}_{t=1}^{\infty}$ a sequence of money supplies $\{M_t\}_{t=1}^{\infty}$, a sequence of money transfers $\{\tau_t\}_{t=1}^{\infty}$, and as sequence of prices $\{p_t\}_{t=1}^{\infty}$, given $M_0$, such that

1. $(c_t^t, c_{t+1}^t)$ and $m_t$ maximize agents' utility subject to budget constraints, given $p_t, p_{t+1}$, and $\tau_t$;
2. money supply grows at a constant rate $z$;
3. government budget constraint holds; and
4. markets clear, $(p_t M_t = L_t p_t m_t)$.

for all $t = 1, 2, \ldots$.

# Non-monetary equilibrium

- In this economy, there always exists a non-monetary equilibrium,
  $\Rightarrow$ a competitive equilibrium where $p_t = 0$ for all $t$, (money has no value) .

- In non-monetary equilibrium

$$(c_t^t, c_{t+1}^t) = (y, 0) \quad \text{and} \quad \tau_t = 0 \qquad \text{for all } t.$$

- For $p_t = 0$, this allocation
  - is feasible,
  - solves utility maximization of an agent,
  - clears the asset (money) and goods markets.

- This allocation is, however, Pareto dominated by allocation $(c_y^*, c_o^*)$:
  - $u(c_y^*, c_o^*) \geq u(y, 0)$,
  - $c_o^* > 0$.

# Absence of double-coincidence-of-wants

- In the absence of money, there is no trade.
- Recall that endowment profile is $(y, 0)$.
- There is absence-of-double-coincidence of wants:
  - an agent born in period $t$ wants to trade period $t$ consumption goods for period $t+1$ consumption goods, but
  - no agent wants to trade period $t+1$ consumption goods for period $t$ consumption goods.

# Monetary equilibria

- Consider equilibria with $p_t > 0$ for all $t$.
- Young agents solve

$$\max_{c_t^t, c_{t+1}^t, m_t} u(c_t^t, c_{t+1}^t)$$

subject to

$$c_t^t + p_t m_t \leq y \qquad c_{t+1}^t \leq p_{t+1} m_t + \tau_{t+1}.$$

taking $p_t$ and $\tau_t$ as given.

- This can be rewritten as

$$\max_{m_t} u(y - p_t m_t, p_{t+1} m_t + \tau_{t+1}).$$

- The first-order condition for an interior optimum is

$$-p_t u_1(y - p_t m_t, p_{t+1} m_t + \tau_{t+1}) + p_{t+1} u_2(y - p_t m_t, p_{t+1} m_t + \tau_{t+1}) = 0$$

where $u_i(c_t^t, c_t^{t+1}) = \frac{\partial u(c_t^t, c_t^{t+1})}{\partial c_t^{t+i-1}}$.

# Real quantity of money

- Let $q_t$ denote real per generation quantity of money:

$$q_t = \frac{p_t M_t}{L_t} \ .$$

- Money market clearing becomes

$$q_t = p_t m_t \ .$$

- Government budget constraint, $L_t \tau_{t+1} = p_{t+1}\left(M_{t+1} - M_t\right)$, implies

$$c_t^{t+1} = p_{t+1} m_t + \tau_{t+1} = p_{t+1}\left(\frac{M_t}{L_t} + \frac{M_{t+1}}{L_t} - \frac{M_t}{L_t}\right) = (1+n) q_{t+1} \ .$$

- Moreover, as

$$\frac{p_{t+1} M_t}{L_t} = \frac{p_{t+1} M_{t+1}}{L_{t+1}} \frac{M_t}{M_{t+1}} \frac{L_{t+1}}{L_t} = \frac{1+n}{z} q_t \ ,$$

the first-order condition for young agents can be written as

$$-q_t u_1(y - q_t, (1+n) q_{t+1}) + q_{t+1} \frac{1+n}{z} u_2(y - q_t, (1+n) q_{t+1}) = 0 \ .$$

# Monetary equilibria

- This is a first-order difference equation in $q_t$.
- Its solution yield the equilibrium sequence of real per generation quantity of money, $\{q_t\}_{t=1}^{\infty}$.
- Using

$$p_t = \frac{q_t N_t}{M_t} \ ,$$

we can solve for price sequence $\{p_t\}_{t=1}^{\infty}$.

- The sequence of money transfers $\{\tau_t\}_{t=1}^{\infty}$ is given by

$$\tau_t = (1+n)q_t \left(1 - \frac{1}{z}\right)$$

while the sequence of consumptions can be determined from

$$\left(c_t^t, c_{t+1}^t\right) = \left(y - q_t, (1+n)q_{t+1}\ \right) \ .$$

# Stationary monetary equilibrium

- Consider a stationary monetary equilibrium, i.e. competitive equilibrium in which

$$q_t = q \qquad \text{for all } t.$$

- The equilibrium can be obtain by solving the first-order condition,

$$u_1(y-q,(1+n)q) = \frac{1+n}{z} u_2(y-q,(1+n)q) \ .$$

- If $z = 1$, i.e. money supply is constant, this equation equals

$$u_1(c_y^*, c_o^*) = (1+n) u_2(c_y^*, c_o^*)$$

with $c_y^* = y - q$ and $c_y^* = (1+n)q$.

$\Rightarrow$ The stationary monetary equilibrium under fixed money supply is Pareto optimal.

# Inflation

- Using equation

$$p_t = \frac{q_t N_t}{M_t} \ ,$$

we can determine the gross inflation rate as

$$\pi_{t+1} = \frac{p_t}{p_{t+1}} = \frac{q_t}{q_{t+1}} \frac{z}{1+n} \ .$$

- In the stationary equilibrium

$$\pi^* = \frac{z}{1+n}$$

and the equilibrium is Pareto optimal regardless of the inflation rate.

- In fact, for any $z \leq 1$, stationary monetary equilibrium is Pareto optimal.

# Example 1

- Assume that $u(c_y, c_o) = \ln c_y + \ln c_o$.
- The condition for monetary equilibrium

$$-q_t u_1(y - q_t, (1+n)q_{t+1}) + q_{t+1}\frac{1+n}{z}u_2(y - q_t, (1+n)q_{t+1})$$

  becomes

$$\frac{q_t}{y - q_t} = \frac{1}{z}.$$

- The solution is

$$q_t = \frac{y}{1+z}.$$

  - The stationary monetary equilibrium is the unique equilibrium.
  - The consumption allocations are

$$(c_t^t, c_{t+1}^t) = \left(\frac{zy}{1+z}, \frac{(1+n)y}{1+z}\right)$$

  $c_t^t$ increases with money growth rate; $c_{t+1}^t$ decreases with $z$.

# Example 2

- Assume that $u(c_y, c_o) = c_y^{\frac{1}{2}} + c_o^{\frac{1}{2}}$.
- The condition for monetary equilibrium for this utility equals

$$q_{t+1} = \frac{q_t^2 z^2}{(y - q_t)(1+n)} .$$

- There are multiple solution to this equation.
- The stationary monetary equilibrium is

$$q_t = \frac{(1+n)y}{1+n+z^2} .$$

- There exists also a continuum of equilibria indexed by $q_1$,

$$q_1 \in \left(0, \frac{(1+n)y}{1+n+z^2}\right) ,$$

  such that

$$\lim_{t\to\infty} q_t = 0$$

  - nonstationary monetary equilibria converging to the nonmonetary equilibrium

# Incomplete Credit Markets and Monetary Policy

Costas Azariadis, James Bullard, Aarti Singh, and Jacek Suda

# Pre-crises monetary policy analysis

- Monetary policy models in central banks
  - Nominal rigidities in prices as key frictions
  - Woodford (2003)

- Pre-crisis consensus for monetary policy
  - optimal to target inflation
  - short-term interest rate and Taylor rule

# Post-crises monetary policy analysis

- The 2007-2009 financial crisis increased attention to credit markets.
- Emphasis on credit market frictions
    - Bernanke, Gertler, Gilchrist (1999), Kiyotaki and Moore (1997)
    - Gertler and Karadi (various), Gertler and Kiyotaki, ...
- New targets for optimal monetary policy
    - price level targeting
    - nominal GDP targeting
- New tools for monetary policy
    - forward guidance, quantitative easing, ...

# This paper: the return of credit markets

- We study an economy with a large private credit market essential to good macroeconomic performance.
  - Main friction: Non-state contingent nominal contracting (NSCNC).
  - The role of monetary policy: keep the large credit market complete and functioning properly.

- When certain shocks hit the economy, the ZLB will be encountered.

- **Main question**:
  - How can the monetary authority maintain a smoothly operating credit market when the ZLB threatens?

# What we do

- Stylized DSGE endowment overlapping generations model with
  - segmented markets: credit sector (participants) and cash sector (non-participants),
  - non-state contingent nominal contracts in credit markets (NSCNC),
  - uncertain labor productivity growth.
- Policymaker (central bank)
  - cares mostly about the credit sector,
  - wants to substitute for the missing state-contingent contracts by choice of the price level,
  - faces, for certain shocks, the threat of the zero lower bound,

# What we find

- Counter-cyclical price level restores optimal risk sharing in the credit market.
  - Optimal monetary policy looks like "nominal GDP targeting"
  - Koenig (2013) and Sheedy (2014)
- When ZLB is expected to bind
  1. higher than usual price level restores complete market allocation,
  2. a set of loans to credit market, "quantitative easing" programme, brings complete market allocation.
- Some drawbacks:
  1. Price level increase harms the cash-using agents.
  2. QE is quasi-fiscal, depriving the fiscal authority of real resources.
- No role for forward guidance.

# ENVIRONMENT

# Segmented markets

- Quarterly $T+1$-period DSGE life cycle *endowment* economy with segmented markets.
- Two assets: *private* debt (consumption loans) and currency.
- Two types of households: *participants* and *non-participants*.
    - Participants can hold either asset, but in the stationary equilibria we study they will not hold currency as it is dominated in rate of return.
    - Non-participants can only hold currency.
- Other assumptions:
    - Within-cohort agents are identical, no population growth, inelastic labor supply, time-separable log preferences, no discounting, no capital, no default, flexible prices, no borrowing constraints.

# Key friction: NSCNC

- Financial markets are incomplete.
- Loan repayments are in nominal terms and are not contingent on future income realizations.
- NSCNC friction is important:
  - Sheedy (2014): both sticky price and NSCNC frictions are present; argues that the NSCNC friction is the more important of the two in a calibrated case.
  - Garriga, Kydland, and Sustek (2013): consider the effect of non-state contingent nominal contracting in housing markets. Quantitatively significant effects of NSCNC.

# Stochastic structure

- Exogenous real wage $w(t)$ follows

$$w(t+1) = \lambda(t, t+1) w(t), \tag{1}$$

with labor productivity growth process given by

$$\lambda(t, t+1) = (1-\rho)\lambda + \rho\lambda(t-1, t) + \sigma\eta(t+1) \tag{2}$$

where $\lambda > 1$ is the average growth rate, $\eta(t+1) \sim N(0, 1)$.

- We assume that $\sigma$ and $\rho$ are chosen such that encounters with the zero lower bound are relatively rare and relatively shallow.

# Timing protocol

- At the beginning of date $t$, agents enter the period with nominal contracts set in $t-1$, $R^N(t-1,t)$
- Nature moves first and chooses $\eta(t)$ and, hence, $\lambda^r(t-1,t)$, which implies value for $w(t)$.
- The policymaker moves next and chooses the price level $P(t)$ .
- Households then decide how much to consume and save/borrow at $R^N(t,t+1)$.

# Life-cycle productivity

- All agents are endowed with an identical productivity profile over their lifetime.
  - The profile begins at zero, rises to a peak at the middle period of life, and then declines to zero.
  - The productivity profile is symmetric.
- Agents supply labour inelastically in the labor market at the competitive wage.

# Productivity profile

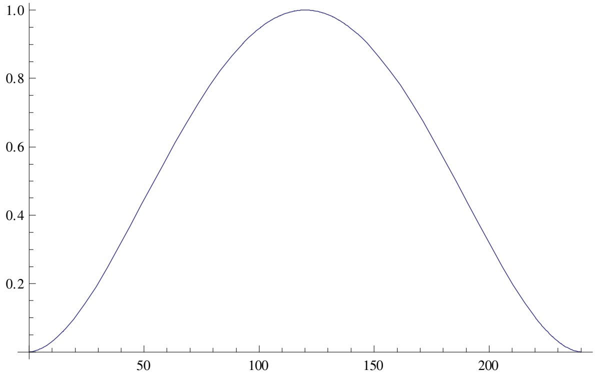


Figure: A schematic productivity endowment profile for credit market participant households. The profile is symmetric and peaks in the middle period of the life cycle.

# Participant households problem

- Agent born at date $t$ maximizes life-time utility

$$E_t \sum_{j=0}^{T} \ln c_t(t+j)$$

subject to

$$c_t(t+j) + \frac{a_t(t+j)}{P(t+j)} = e_j w(t+j) + \frac{R^N(t+j-1, t+j)a_t(t+j-1)}{P(t+j)}$$

- Total income in the credit sector at date $t$ is $w(t) \sum_{j=0}^{T} e_j$.
- Euler equation for any generation $j$ at date $t$ is given by

$$R^N(t, t+1)^{-1} = E_t \left[ \frac{c_{t-j}(t)}{c_{t-j}(t+1)} \frac{P(t)}{P(t+1)} \right].$$

# Non-participants

- Completely precluded from credit markets.
- *No life cycle aspect* to productivity or consumption.
  - Inactive in the first period 0.
  - Productivity endowment $\gamma$ is "small"
  - Earn $\gamma w(t+s)$ in odd-dated stages, $s = 1, 3, 5, ..., T-1$.
  - Consume in even-dated stages $\ln c_t(t+s)$, $s = 2, 4, 6, ..., T$.
  - Real money demand given by

$$h^d(t) = \frac{T}{2}\gamma w(t)$$

  where $h^d(t) = H(t)/P(t)$.
- Non-participants work only intermittently and save all income by holding currency.

# Currency provision

- Central bank prints currency and sells it to non-participant households.
- Currency stock at date $t$

$$H(t) = \theta(t-1,t)\,H(t-1).$$

- The central bank completely controls the date $t$ price level via the gross growth rate of currency creation, $\theta(t-1,t)$ , which is

$$\frac{T}{2}\gamma w(t)P(t) = \theta(t-1,t)\,\frac{T}{2}\gamma w(t-1)P(t-1)$$

$$\theta(t-1,t) = \frac{P(t)}{P(t-1)}\frac{w(t)}{w(t-1)}.$$

- By choosing $\theta(t-1,t)$, central bank determines $P(t)$.

# Nominal interest rate

- Participant households contract by fixing the nominal interest rate one period in advance.
- The non-state contingent nominal interest rate, the contract rate, is given by
$$R^N(t, t+1)^{-1} = E_t\left[\frac{c_{t-j}(t)}{c_{t-j}(t+1)}\frac{P(t)}{P(t+1)}\right].$$
- This rate depends on the expected rate of consumption growth and the expected rate of inflation.
- We study stationary equilibria in which the zero lower bound is never breached.

# The fiscal authority

- The scope of action for the fiscal authority is limited.
  - It interacts only with the central bank in normal times.
- The fiscal authority does not levy taxes at any time.
- The fiscal authority is the only agent with a storage facility

$$x(t+1) = R(t,t+1)x(t) + \frac{H(t+1) - H(t)}{P(t+1)}$$

where real rate of return to storage is the same as the real rate of return in the credit sector.

  - Record-keeping device—how much real seigniorage revenues received from CB.

# The central bank

- The central bank supplies currency to the cash sector of the economy, which determines $P(t)$.
- The policy rule for $P(t)$ will be designed to complete credit markets.
- The central bank does not consume and is independent, which means it interacts with other agents at market prices.
- Seigniorage earned by the central bank is lent to the fiscal authority in return for debt paying the same rate of return as in the credit market.
- The fiscal authority puts this consumption into storage.
- The debt issued can be continually refinanced.

# The central bank mandate

- The central bank has a hierarchical mandate.
- **First** and foremost, the central bank mandate calls for smoothly operating credit market, a form of "financial stability."
- **Secondarily**, the central bank is expected to maintain an exogenously given inflation target, so as not to harm the cash users in the economy too much in pursuit of the first goal.
- We assume an inflation target of zero for convenience.

# Stationary equilibria

- Let $t \in (-\infty, +\infty)$ .
- We only consider stationary equilibria under perfectly credible policy rules governing $P(t)$ .
- Stationary equilibrium is a sequence $\left\{R^N(t, t+1), P(t)\right\}_{t=-\infty}^{+\infty}$ such that households solve their optimization problem, the policymaker credibly adheres to a stated policy rule governing $P(t)$ and markets clear.
- Key condition is clearing of aggregate asset holding

$$\frac{A(t)}{P(t)} = \frac{\sum_{j=0}^{T-1} a_{t-j}(t)}{P(t)} = 0, \qquad \forall t.$$

# Balanced Growth

# Non-stochastic net asset holding

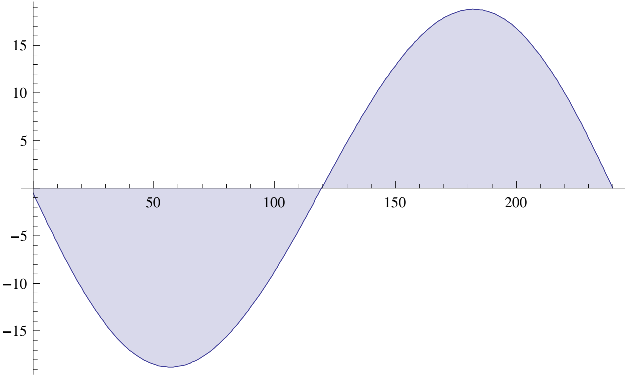


Figure: Net asset holding by cohort along the balanced growth path. Borrowing, the negative values to the left, peaks at stage 60 of the life cycle, roughly age 35, while positive assets peak at stage of life 120, roughly age 65.

# Non-stochastic consumption

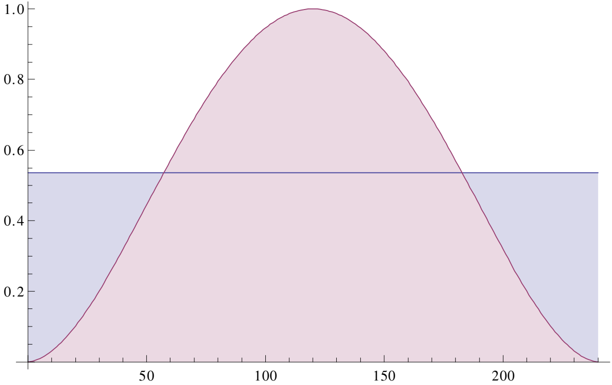


Figure: Schematic representation of consumption, the flat line, versus income, the bell shaped curve, by cohort along the complete markets balanced growth path with $w(t) = 1$. The private credit market completely solves the point-in-time income inequality problem.

# Key feature of the non-stochastic steady state

- In equilibrium with zero net inflation (price stability)

$$R = \lambda.$$

- All income earned within any period is divided equally among all participants alive in the economy at that time.
  - *Households have an "equity share" in the economy*.
  - Consumption is flat.
- In cash market, $\theta = \lambda$

$$R^N = \lambda > 1,$$

so the *net nominal interest rate* would always be positive and money dominated by private debt in rate of return.

# Complete markets

# Complete markets without NSCNC

- Now allow aggregate shocks, maintaining the price stability policy $P(t) = 1 \; \forall t$.
- Set the NSCNC friction aside *for this slide only*.
- In equilibrium,

$$R(t-1,t) = R^N(t-1,t) = \lambda(t-1,t),$$

and

$$c_{t-i}(t) = \frac{w(t) \sum_{i=0}^{T} e_i}{T+1}.$$

  - Participants consume equal amounts of available output in the credit sector.
  - Consumption and asset holdings fluctuate from period to period *but in proportion to the value of $w(t)$*.

# Monetary Policy with NSCNC

- Assume the NSCNC friction and consider $\sigma$ close to zero.
- Borrowers and lenders contract on $R^N(t-1,t)$ at date $t-1$ which is non-state contingent.

$$R^N(t-1,t)^{-1} = E_{t-1}\left[\frac{c_{t-1}(t-1)}{c_{t-1}(t)}\frac{P(t-1)}{P(t)}\right].$$

- The *countercyclical price level rule*,

$$\begin{aligned} P(t) &= \frac{E_{t-1}[\lambda(t-1,t)]}{\lambda(t-1,t)}P(t-1) \\ &= \frac{(1-\rho)\lambda + \rho\lambda(t-2,t-1)}{(1-\rho)\lambda + \rho\lambda(t-2,t-1) + \rho\eta(t)}P(t-1), \end{aligned}$$

delivers complete markets allocation.

# The nature of complete markets policy

- Households consume equal amounts of available output in the credit sector.
- Consumption and asset holdings fluctuate from period to period, but in proportion to the value of $w(t)$ .
- This policy involves countercyclical price level movements.
    - Real debt payments higher when income is higher and vice versa.
- Inflation is high when growth is low, and inflation is low when growth is high.
    - On average, an inflation target could still be maintained.
- Rule can be implemented *via* a simple money growth rule.
- It works well provided the ZLB is never encountered.
- It can be interpreted as nominal income targeting.

# Interpretation as nominal GDP targeting

- At date $t$, nominal GDP equals

$$Y^{n}(t) = P(t)\, w(t) \left[ \frac{T\gamma}{2} + \sum_{i=0}^{T} e_i \right].$$

- In the non-stochastic case, (with $P(0) = w(0) = 1$),

$$Y^{n,\star}(t) = \lambda^{t} \left[ \frac{T\gamma}{2} + \sum_{i=0}^{T} e_i \right],$$

$$Y^{n,\star}(t+1) = \lambda P(t)\, w(t) \left[ \frac{T\gamma}{2} + \sum_{i=0}^{T} e_i \right].$$

- Nominal GDP at date $t+1$:

$$Y^{n}(t+1) = \left[ (1-\rho)\lambda + \rho\lambda(t-1,t) \right] P(t)\, w(t) \left[ \frac{T\gamma}{2} + \sum_{i=0}^{T} e_i \right].$$

# ZERO LOWER BOUND

# Encounters with the zero lower bound

- The economy grows over time at the average gross rate $\lambda$ and the inflation target is zero.
- The zero lower bound is encountered when expected net consumption growth is negative,

$$R^n(t, t+1) = E_t \lambda(t, t+1) \leq 1.$$

  - At date $t$, there is a large negative shock, $\eta(t) << 0$.
  - Agents expect the net nominal interest rate to be below the zero if

$$\lambda(t-1, t) \leq \frac{1-(1-\rho)\lambda}{\rho}.$$

- If the nominal interest rate were allowed to be zero, the saver segment of participant households would want to hold currency.
- If there is no additional intervention by CB, credit market is disrupted.

# Two policy options

- The central bank in this situation would wish to keep the nominal interest rate positive.
- The definition of the nominal interest rate suggests two methods.
- One is to promise a one-time increase in the price level for the following period sufficient to keep the nominal rate positive.
- The other is to promise that the consumption growth rate in the credit sector will increase in order to keep the nominal rate positive.
- Either approach would have to be part of a credible commitment to a policy rule.

# Policy 1: Higher inflation

- Let the price level policy rule equal

$$P(t+1) = \begin{cases} \frac{E_t[\lambda(t,t+1)]}{\lambda(t,t+1)} P(t) & \text{if} \quad E_t\lambda(t,t+1) > 1 \\ \frac{E_t[\lambda(t,t+1)](1+\vartheta_p(t+1))}{\lambda(t,t+1)} P(t), & \text{if} \quad E_t\lambda(t,t+1) \leq 1 \end{cases}$$

  with $\vartheta_p(t+1) > 0$ such that $(1+\vartheta_p(t+1))E_t[\lambda(t,t+1)] = 1^+$.

- Then, at date $t+1$:
  - $R^N(t,t+1) = (1+\vartheta_p(t+1))E_t[\lambda(t,t+1)] = 1^+$.
  - $R(t,t+1) = \lambda(t,t+1)$.
  - Consumption moves with income.
- One-time increase in the price level.
- Complete market allocation for the credit users.
- Drawback: Cash-using households affected.

# Policy 2: Balance sheet policy

- The central bank could make loans to participant households in proportion to earned income.
- This would mean that total income in the sector would be proportionately higher during the period of the loans.
- Such a program would require a one-time transfer of real resources to the credit sector.
  - Alternatively, the loans could be rolled over, but forgiven at the end of life.
- Credit sector participants would see this as additional income, and would again consume exactly equal amounts, maintaining complete market allocations.
- Drawback: Resources are diverted from the fiscal authority.

# Policy 2: Balance sheet policy

- Assume price rule is unchanged

$$P(t+1) = \frac{E_t[\lambda(t,t+1)]}{\lambda(t,t+1)} P(t).$$

- Let the cohort-specific loan/consumption transfer at date $t+1$, $\tau_j(t+1)$, be proportional to income

$$\tau_j(t+1) = (1+\vartheta_{qe}(t+1))w(t+1)e_j,$$

  where $\vartheta_{qe}(t+1) > 0$ is such that $(1+\vartheta_{qe}(t+1))E_t[\lambda(t,t+1)] = 1^+$.

- *Expected* consumption equals

$$E_t c_{t-j}(t+1) = c_t(t)(1+\vartheta_{qe}(t+1))E_t[\lambda(t,t+1)].$$

- Then the nominal interest rates are

$$R^N(t,t+1) = (1+\vartheta_{qe}(t+1))E_t[\lambda(t,t+1)] = 1^+$$

$$R^N(t+1,t+2) = \frac{E_{t+1}[\lambda(t+1,t+2)]}{(1+\vartheta_{qe}(t+1))}$$

- The nominal interest rate remains close to 1 following the QE intervention.

# Conclusion

# Conclusion

- The model features an important credit market and the friction is NSCNC.
- Restoring market completeness requires counter-cyclical price level.
- This paper suggests two methods of conducting monetary policy when the ZLB threatens.
  1. Higher than *usual* price level.
  2. Carefully designed balance sheet approach.
- These two methods look somewhat different from what has been done in the last 5 years, even though the desire behind many actual policy choices has been to help credit markets perform better.

# Reaching ZLB

- Assume that economy is in the steady-state at date $t-1$, $\lambda(t-2, t-1) = \lambda$.
- For $E_t \lambda(t, t+1) \leq 1$, it must be the case that

$$\eta(t) \leq \frac{1-\lambda}{\sigma\rho}.$$

- Since $\eta(t) \sim N(0,1)$, the probability of ZLB at date $t+1$ equals

$$P\left(\eta(t) \leq \frac{1-\lambda}{\sigma\rho}\right) = \Phi\left(\frac{1-\lambda}{\sigma\rho}\right).$$

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# Staying at ZLB

- Let the economy at date $t-1$ be at the zero-lower bound, $E_{t-1}\lambda(t-1,t)=1$, or $\lambda(t-2,t-1)=\frac{1-(1-\rho)\lambda}{\rho}$.
- The economy will stay at the bound at date $t$ when

$$\eta(t) \leq \frac{(1-\rho)(1-\lambda)}{\sigma\rho}.$$

- This will happen with probability $\Phi\left(\frac{(1-\rho)(1-\lambda)}{\sigma\rho}\right)$.

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