Advanced Macroeconomics
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Problem Set 1

Due date: Wednesday, 10 May, 16:10

1. **Question 1**

   Consider a simple version of New-Keynesian DSGE model analyzed by Woodford (1999). The reduced form system can be written as

$$y_t = E_t y_{t+1} - \sigma^{-1}\left[r_t - E_t \pi_{t+1}\right] + \sigma^{-1} r_t^n \quad (1)$$

$$\pi_t = \kappa y_t + \beta E_t \pi_{t+1} \quad (2)$$

$$r_t = \phi_\pi \pi_t + \phi_y y_t \quad (3)$$

   y_t denotes the output gap (i.e. $y_t = \hat{Y}_t - \hat{Y}_t^s$, where $\hat{Y}_t$ is the actual level of output and $\hat{Y}_t^s$ is potential level of output, expressed in percentage deviations from the steady state); π_t is the inflation rate (expressed as a deviation from the target rate π^*); and r_t is the nominal interest rate (expressed, again, as deviation of short term nominal interest rate from the target rate, r^*). r_t^n denotes the "natural rate of interest" and is assumed to be an exogenous stochastic term that follows an AR(1) process

$$r_t^n = \rho r_{t-1}^n + \epsilon_t, \qquad \epsilon_t \sim N(0, \sigma_\epsilon^2), \quad |\rho| < 1. \quad (4)$$

   Parameters of the model (σ, β, κ) are structural parameters given by the full model. In particular, σ is the elasticity of intertemporal substitution of the representative household, β is the household's discount factor, and κ is related to the price stickiness in the firm's problem (see Woodford (2003) or Rottemberg and Woodford (1999) for the detailed description of the model). Parameters ϕ_π and ϕ_y describe monetary policy response to deviation of inflation and output from their targets. In particular, if $\phi_\pi > 1$ (also known as "Taylor Principle") we say that monetary policy is *active*, and if $\phi_\pi < 1$, monetary policy is considered *passive*.

   Equation (1) is derived from the household optimization problem and is often referred to IS curve[1]; equation (2) is derived from the firms' pricing problem and is often referred to a New-Keynesian Philips curve; whereas equation (3) is an ad-hoc monetary policy Taylor-type feedback rule.

   (a) Use equation (3) to write the system as two equations with two endogenous variables (treat r_t^n as exogenous variable) and express the system as

$$E_t x_{t+1} = A x_t + E r_t^n. \quad (5)$$

   Report A and E.

[1] In the underlying structural model there is no capital and $Y_t = C_t + G_t$, with G_t stochastic and exogenous.

(b) Given the results in Blanchard and Khan (1980), what is the condition for the existence and uniqueness of equilibrium in this model?

(c) Write down these conditions in terms of structural parameters of the model.

(d) Set parameter value as in Rotemberg and Woodford (1999), i.e. $\beta = 0.99$, $\sigma = 0.16$, and $\kappa = 0.024$. Does monetary policy stance (i.e. ϕ_y and ϕ_π) play a role for the existence and/or uniqueness of equlibrium ?

(e) Consider the case where the central bank does not respond to output gap when setting short term nominal interest rate, i.e. $\phi_y = 0$. For $\kappa > 0$ how much nominal interest rate should be raised if inflation is above the target? How would we call this type of monetary policy?

(f) Assume that central bank and/or monetary authorities observe inflation and output only with a lag. The monetary policy rule will now be of form

$$r_t = \phi_\pi \pi_{t-1} + \phi_y y_{t-1}, \tag{6}$$

instead of the one given by equation (3). Rather than substituting for r_t in equation (1), move equation (6) one period forward. Now the system consists of 3 equations, (1)–(2) and (6), with 3 endogenous variables. Write down the system as

$$E_t x_{t+1} = A x_t + E r_t^n. \tag{7}$$

Given the results in Blanchard and Khan (1980), what is the condition for the existence and uniqueness of equilibrium in terms of matrix A of this model?

2. **Question 2**

Consider a simplified version of the RBC model we discussed in the class, Ramsey's (1928) optimal growth model.[2] The household problem is then to maximize utility

$$E_t \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}) \tag{8}$$

subject to the budget constraint

$$c_t + i_t = r_t k_t + Div_t \tag{9}$$

and the capital accumulation rule

$$k_{t+1} = (1-\delta) k_t + i_t. \tag{10}$$

We will assume that

$$u(c_t) = \ln c_t.$$

A representative firm uses capital, rented from households, to produce a unique good y_t and to maximize profits:

$$Div_t = y_t - r_t k_t \tag{11}$$

[2] It is similar to assuming that labor is supplied inelastically, i.e. $l_t = 1$.

subject to technology

$$y_t = z_t k_t^{\alpha}, \tag{12}$$

where z_t denotes aggregate productivity, that follows and AR(1) process

$$z_t = exp(\epsilon_t) z_{t-1}^{\rho}, \qquad \epsilon_t \sim N(0, \sigma_{\epsilon}^2). \tag{13}$$

Assume goods market clears.

(a) Derive first order conditions.

(b) Compute steady state.

(c) Log-linearize the equations.

(d) Express the model in Sim's (2001) notation

$$\Gamma_0 x_{t+1} = \Gamma_1 x_t + C + \Psi v_{t+1} + \Pi \eta_{t+1}.$$

where η_{t+1} is vector of expectation errors, v_{t+1} is a vector of exogenous shocks, and x_t a vector of endogenous variables.

(e) Parameterize the model with the following values

$$\begin{aligned} \beta &= 0.99 \\ \alpha &= 0.67 \\ \delta &= 0.025 \\ \rho &= 0.979 \\ \sigma_e &= 0.072 \end{aligned}$$

Using the Sims' code `gensys` available at `http://sims.princeton.edu/yftp/gensys/` determine whether the equilibrium exists and is unique. Attach the code you wrote.

Hint: You have to write a short simple code that will compute the answer ro the the existence and uniqueness question. Note that you need all files from the Sims' website to be in the same directory as (or at least visible to) your own Matlab file.

To use `gensys` code you need to specify the input matrices. They are given by the equation

$$\Gamma_0 x_{t+1} = \Gamma_1 x_t + C + \Psi v_{t+1} + \Pi \eta_{t+1}.$$

In Matlab, specify the parameters value (as above), e.g.

```
delta = 0.025;
```

and steady states

```
Rstar = 1/beta;
```

Then specify the matrices and vectors given in the equation above, e.g. if you have 5 variables in x_t,

```
c = zeros(5,1);
```

(since we expressed the model in terms of deviation from the steady state, vector of constants if equal to zeros),

```
g0 =  [ 1 0 0 -alpha -1;
        0 0 0 0 1 ;
        ....
        0 0 0 0 1 0 ];
```

for all matrices. Once you specified all necessary matrices `g0,g1,c,psi,pi`, use the Sims procedure/function `gensys`:

```
[G1,C,impact,fmat,fwt,ywt,gev,eu] = gensys(g0,g1,c,psi,pi);
```

The existence and uniqueness conditions are in variable `eu` that `gensys` outputs. `eu` is a vector 2×1: if `eu(1)=1` the equilibrium exists, and if `eu(2)=1` it is unique. For more details see description inside `gensys.m` and associated `pdf` on the website.

References

Blanchard, O. and C. Kahn (1980), "The Solution of Linear Difference Models under Rational Expectations," *Econometrica* 48(5), 1305–1311.

Rottemberg, J. and M. Woodford (1999), "Interest Rate Rules in an Estimated Sticky Price Model," NBER Chapters, in: *Monetary Policy Rules*, 57–126, National Bureau of Economic Research, Inc.

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Woodford, M. (1999), "Optimal Monetary Policy Inertia," *The Manchester School* 67(1), 1–35.

Woodford, M. (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.