Advanced Macroeconomics
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# Problem Set 3

Due date: Wednesday, 14 June, 23:59

1. **Question 1** (OLG)

   Consider the following overlapping generations model. Time is indexed by $t = 0, 1, \ldots$. In period $t$, $L_t$ two-period-lived consumers are born. Assume

   $$L_t = L_0(1+n)^t$$

   with $L_0 > 0$. When young, each of these consumers is endowed with one unit of labor, and has preferences given by

   $$u(c_t^y, c_{t+1}^o) = v(c_{t+1}^o)$$

   with $v(\cdot)$ strictly increasing. There is a group of one-period-lived old agents alive at date 0 who are collectively endowed with $K_0 > 0$ units of capital. These one-period-lived consumers maximize consumption at date 0. The representative firm has a production technology given by

   $$Y_t = K_t^\alpha L_t^{1-\alpha}, \qquad 0 < \alpha < 1.$$

   (a) Determine consumption of the young, consumption of the old, and the capital-labor ratio in the optimal steady state.

   (b) Determine consumption of the young, consumption of the old, and the capital-labor ratio in a competitive equilibrium steady state. How does this differ from the optimal steady state in part (a), and why?

   (c) Now, suppose the government issues $B_{t+1} > 0$ bonds in period $t$, with $B_{t+1} = bL_t$ for all $t \geq 0$. Each young agent is taxed lump-sum ($\tau_t$) so that the government can finance any interest payments on the debt that cannot be financed with the current bond issue. Determine the value for $b$ that implies that the optimal steady state is achieved as a competitive equilibrium steady state. Is $b$ positive or negative? Explain your results.

2. **Question 2** (OLG with growth)

   Assume the same setup as in the previous question, with the following exceptions. The production function is now given by

   $$Y_t = K_t^\alpha \left(z_t L_t\right)^{1-\alpha}, \qquad 0 < \alpha < 1,$$

   with

   $$z_t = (1+\lambda)^t z_0.$$

   Assume $z_0 > 0$ and $\lambda > -1$. This specification implies that there is labor-augmenting technical change.

(a) Determine the steady state for this economy. Here, it will help to express it in terms of that $k_t \equiv \frac{K_t}{z_t L_t}$. Note that $k_t$ converge to a constant in the steady state.

(b) What are the growth rates of aggregate output, the aggregate capital stock, and aggregate consumption in the steady state?