Advanced Macroeconomics
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Problem Set 2

Due date: Monday, 5 June, 12:10

1. **Question 1** (Stochatic RBC: Guess and verify)
   Show that in the stochastic growth model discussed in lecture 7 if there is less than 100%, depreciation the solution to the policy function given by $k' = \phi(A, k) = \lambda A k^{\alpha}$ fails.

2. **Question 2** (Deterministic Dynamic Programming)
   Consider the social planner problem of maximizing utility

$$\sum_{t=0}^{\infty} \beta^t U(c_t)$$

   subject to a resource constraint

$$c_t + k_{t+1} = f(k_t) + (1-\delta)k_t, \qquad k_0 \text{ given.}$$

   (a) Provide a dynamic programming representation of this problem.

   (b) Let the period utility function be

$$U(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}, \quad \sigma > 0$$

   and let the production function be

$$f(k) = Ak^{\alpha}, \qquad 0 < \alpha < 1$$

   and suppose the parameter values are

$$\beta = 0.95, \quad \alpha = 0.33, \quad A = 1, \quad \delta = 0.04, \quad \sigma = 2.00.$$

   Solve for steady state consumption c^*, capital stock k^*, and the value of life-time utility for steady state consumption

$$V^* = V(k^*) = \sum_{t=0}^{\infty} \beta^t U(c^*).$$

   (c) Using/modifying the example code `dynamicProgramming_deterministicRBC.m`

      i. Construct a grid for capital $k \in K = \{k_0, k_1, \ldots, k_{n_K}\}$ with $k_0 = 0$ and $k_{n_K} = 5 * k^*$ with $n_K = 1000$ evenly spaced elements.

      ii. Solve the dynamic programming problem on this discrete state space by value function iteration.

iii. Plot the value function that is a fixed point of the Bellman equation and plot the associated policy function.

iv. Show/write how different values of β affect the number of iterations that you need for convergence.

v. What happens when you change the number of elements in the grid, n_K?

vi. What happens when you change the grid for capital to $k \in \tilde{K} = \{k_0, k_1, \ldots, k_{n_K}\}$ with $k_0 = 0$ and $k_{n_K} = \bar{k}$, where $\bar{k}$ is the solution to

$$k = Af(k) + (1 - \delta)k$$

?

vii. What happens when you change the starting point from $V_0(k) = 0$ to $V_0(k) = V^*$?

Attach relevant figures.