The dangers of macro-prudential policy experiments: initial beliefs under adaptive learning*

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Abstract

The paper studies the implication of initial beliefs and associated confidence on the system's dynamics under adaptive learning. We first illustrate how prior beliefs determine learning dynamics and the evolution of endogenous variables in a small DSGE model with credit-constrained agents, in which rational expectations are replaced by constant-gain adaptive learning. We then examine how discretionary experimenting with new macroeconomic policies is affected by expectations that agents have in relation to these policies. More specifically, we show that a newly introduced macro-prudential policy that aims at making leverage counter-cyclical can lead to substantial increase in fluctuations under learning, when the economy is hit by financial shocks, if beliefs reflect imperfect information about the policy experiment. This is in the stark contrast to the effects of such policy under rational expectations.

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1 Introduction

How to design and then introduce an effective policy? How to properly take incentives into account and how to induce particular actions? What should be the policy response to extreme events? How to address persistent slow-moving trends? When should be a new policy deployed?

These are among myriads of questions that policymakers face. They arise in times of crises (e.g., health, economic, financial, sovereign debt, pension) and during the "normal times". Studies in political science, sociology, social policy, and economics have looked for answerers to these questions. How to design macroeconomic policy and how people respond to it were at the heart of the rational expectations revolution. Rational expectations hypothesis has been a cornerstone of modern macroeconomics for over 40 years. Explicitly modeling expectations in a model addresses main drawbacks of macroeconomic models of 1960s and 1970s: it takes into account that (i) expectations about future matters for today's decision and (ii) agents react to the changes in their economic environment, e.g., taxes. Agents' behavior reflects the expectations about the future path of the economy: regardless of whether its dynamics are driven by a policy or a shock.

While elegant and simple in its nature—being just mathematical expectations—the assumption of rational expectations is informationally quite demanding. It implies not only that agents are individually rational, but also that their beliefs are consistent. The latter implies that agents use the correct probability measure: the subjective distribution they use to forecast unknown variables agrees with the actual distribution of these variables. The rational expectation hypothesis implicitly assumes that agents have a lot of knowledge. For one, the unbiasedness and the consistency of the forecasts imply that agents, be it households or firms, have a very good grasp of the economy: when they make their prediction, they are on average correct and they do not make consistent mistakes.

A support for the rational expectation as the equilibrium concept is offered by the adaptive learning approach. It assumes that even though agents may not know everything and that their subjective distribution may not fully match the objective one, they are able to learn and update their beliefs eventually arriving at the rational expectations. The important features of this approach are (i) how agents learn, (ii) what their initial beliefs are, and (iii) how likely they are to change their mind. If agents learn and update their beliefs what matters is how confident they are in their priors - if they are fairly skeptical they revise their beliefs quickly and these initial beliefs matter only temporarily. If, however, they firmly believe they are correct they are not likely to update revise their priors substantially.

In this paper we study how expectations and the dynamics of the model are affected by initial beliefs. We then examine to what extent it affects macroeconomic policy implementation.

We find that initial beliefs about a policy matter. A lot.

1.1 What we do and findings

We start with a simple variant of an RBC model to show the mechanism through which initial beliefs and confidence affect macroeconomic dynamics. We replace rational expectations agents with econometricians and study how their estimates that describe the economy's perceived law of motion are affected by the prior beliefs and their variance. We then extend the model to allow for richer dynamics and the role for experimenting with macroeconomic policy. Following Pintus and Suda (2019), we use a version of their collateral constraint model. Pintus and Suda (2019) show that the interaction of financial markets and learning could partially explain both the onset and the severity of the crisis. The last global financial crisis brought calls for policy solutions that were not considered or even available before the crisis. Fiscal policy, monetary policy and macro-prudential policy were used to respond to the repercussion of the crisis itself, but it is largely macro-prudential policy that is sought to address some of its underlying factors. For that reason in this paper we focus our attention on macro-prudential policy.[1]

As in the simple model, we assume that agents use the constant-gain adaptive learning instead of the rational expectations. The underlying premise is that households do not have perfect knowledge about the model and its parameters, including policy parameters, so they use historical data to learn about them and for forecasting. Agents behave as econometricians and form expectations about future treating current real-

[1]Although the broad question we address also pertains to unconventional monetary policy, for instance when it was introduced for the first time in the US and in the Eurozone, we abstract from this important dimension due to the lack of a canonical model. In contrast, both the collateral channel and macro-prudential policies targeting leverage are now part of the macroeconomic toolkit, see Millard et al. (2021) for a review.

izations macroeconomic variables as a linear function of their past realizations. As the new data becomes available every period, they update the coefficients of these linear functions every period. Given this structure, agents' expectations depend on the time–varying coefficients that represents agents' beliefs and their current perception of how the economy works. This method has two important implications. First, the initial beliefs (priors) may have lingering effects both for the subsequent evolution of beliefs and for the resulting dynamics of the entire system. Second, the introduction of (i.e., experimenting with) a new policy or a modification of the existing one alter the law of motion and need to be learnt. This might be a lengthy process, especially if the policy change is not properly announced and explained, that yield dynamics and outcomes that are different than not only those under rational expectations, but also those under learning with alternative priors. It creates an entirely new problem for the policy design and for its implementation, namely, how to deal with beliefs that encapsulate imperfect information about the economy and about how policies affect it.[2]

To assess the importance of these two implications we analyze (i) how the assumptions about priors and their variance describing the adaptive learning matter for the parameter estimates and (ii) how the introduction of a new economic policy is affected by the initial perception and learning about that policy. Our results indicate that the dynamics of the economy under adaptive learning is very different from the dynamics under rational expectations. We also find that agents' behavior under adaptive learning is far from uniform. We show that it is the initial beliefs and confidence in them that determines the initial response of the economy and its subsequent dynamics. The less "trust" agents have in their initial beliefs, the bigger the revisions of agents' beliefs and the larger the impact on endogenous variables. Importantly, however, we find that the extent to which the introduction of a new policy can be deemed successful does vary on whether agents account for that in their perceived law of motion.

These results yield important implications for the policy design. An introduction of new policy or a modification of an existing one is most likely to change the existing economy's law of motion. The extent to which agents' perceived law of motion responds to it determines agents' perception of the economy under the new policy and, in consequence, its eventual success. It should be the goal for the policymakers to inform agents how this policy will operate.

Given the importance of the financial markets in the recent global financial crisis we

[2]Although one may get the impression that the issue revolves simply around communication, it goes beyond in a world where both policymakers and the private sector have knowledge about how the economy works that is far from perfect.

consider the case of macro-prudential policy as a simple but important example. Even though we consider a specific case of macro-prudential policy as an example our results and conclusions are general to any policy. This has an immediate repercussion for the viability and success of any experiments involving conventional or unconventional policies.

1.2 Related literature

Our paper relates to several strands of the literature. The closest to out paper is Honkapohja and Mitra (2020), who study the credibility of a newly introduced policy in the model with learning. They consider the case of the introduction of the price level targeting where the credibility varies endogenously over time in response to the relative performance of inflation forecasting. Their focus is, however, on the expectational stability of the system rather than on the dynamics.

The impact of the prior beliefs on the dynamics of a macroeconomic system under learning is analyzed in Bullard and Suda (2016). They study the expectational stability of macroeconomic system with Bayesian learners, who use Bayesian updating rather than recursive least squares or constant gain as adaptive schemes. They show that while the priors and their variance do not affect stability of the system, it can affect the dynamics. Similarly, Cogley and Sargent (2008) and Suda (2018) show that the variance/precision of initial beliefs matter for the dynamics of beliefs and endogenous variables. In particular, they show that the speed of belief updating could affect both asset prices (Cogley and Sargent, 2008) and quantities, like output, capital, or consumption (Suda, 2018). The model there, however, consider learning about a Markov transition matrix with alternative Bayesian learning. Similarly to these papers we on the importance of priors for the subsequent learning process. Not only we look at the evolution of beliefs related to the law of motion of the economy but also study how much these beliefs change. We use more standard version of adaptive learning that assumes less sophistication on the part of the agents. Additionally, our focus is on the importance of these results for the economic policy.

In our model the key relationship is between changes in the leverage, house prices and agents' decisions. Bailey et al. (2019) study the relationship between homebuyers' beliefs about future house price changes and their mortgage leverage choices. They work focus, however, on the role of heterogeneous beliefs in explaining households' financial decisions.

Our main assumption is that agents' expectations may not be the same as full information rational expectations (FIRE). There is a vast literature that questions such assumption.[3] Hey (1994) rejects rational expectations and finds evidence that adaptive expectations have explanatory power for belief dynamics. The impact of non-rational expectations in the DSGE models has attracted some attention as departure from FIRE can bring empirically relevant results. Farhi and Werning (2019) extend the benchmark New-Keynesian model with bounded rationality. They show that it is the combination of the departure from rational expectations with agents' heterogeneity, incomplete and occasionally binding borrowing constraints that can deliver rationalization of the "forward guidance puzzle". Eusepi and Preston (2018) show how incomplete knowledge and learning can make public debt important for inflation - a conclusion that is absent under rational expectations.

Finally, we contribute to vast and growing literature on macro-prudential regulations, see Cerutti et al. (2017) and Millard et al. (2021) for recent surveys. To our knowledge, ours is the first paper that examines the impact of initial beliefs and adaptive learning on the effectiveness of this class of policies.

The rest of the paper is structured as follows. In section 2 we use a simple model with collateral constraint to illustrate how, in the adaptive learning setting, prior beliefs affect expectations, the evolution of beliefs, and the response of the economy to shocks. In section 3 we extend the basic model with the endogenous leverage and introduce the role for macro-prudential policy. Section 4 shows how the introduction of an economic policy designed to make the leverage counter cyclical and to reduce and mute the negative effects of financial shocks may actually lead to the higher volatility and the amplification of such shocks. In section 5 we discuss the implication for the design and the deployment of new macroeconomic policies in general and section 6 concludes.

2 The importance of initial beliefs: simple illustration

We start with a simple framework to illustrate the effects of prior beliefs on the dynamics of the endogenous variables and the path of learning itself. We consider a simple model with a collateral constraint that can be considered a special version of the richer model we use below. We abstract from nominal frictions and, in fact, our model is isomorphic to an RBC model.

[3] See the survey in Manski (2018) on measuring expectations and confronting the empirical evidence with theory of rational expectations.

2.1 Model

The model features two classes of households: borrowers and lenders that differ with respect to the discount factor. A unit mass of borrowers derives the utility from consumption (subject to external habits) and operate a production function that will be the source of their income. In particular, they maximize

$$\max E_0^* \sum_{t=0}^{\infty} \beta^t \frac{(C_t - \rho \bar{C}_{t-1})^{1-\sigma}}{1-\sigma}, \tag{1}$$

where C_t denotes the borrowers' period consumption, $\bar{C}$ is the average consumption, and β is the discount factor of borrower subject to a budget constraint

$$C_t + Q_t(L_{t+1} - L_t) + R_{t-1}B_t \leq Y_t + B_{t+1} \qquad [\lambda_t] \tag{2}$$

and the borrowing constraint

$$R_t B_{t+1} \leq \theta L_{t+1} E_t^* Q_{t+1} \qquad [\phi_t]. \tag{3}$$

Here B_t denotes the bonds/debt, R_t is the gross real interest rate, L_t is the stock of land, Q_t is the price of land, and λ_t and ϕ_t are Lagrange multipliers associated with the budget and the collateral constraints, respectively. The leverage is given by θ which describes what fraction of expected value of land can household use as a collateral. We also allow for the possibility of a non-rational expectation, E_t^*. Output, Y_t, is produced with land only:

$$Y_t = A_t L_t^{\gamma}, \tag{4}$$

where A_t is the stochastic technology described by a simple AR(1) process,

$$\ln A_t = a_t = \rho_a a_{t-1} + \varepsilon_t. \tag{5}$$

Lenders choose the consumption, $\tilde{C}$, lending $\tilde{B}$, and the land holdings, $\tilde{L}$, to maximize their life-time utility

$$\max E_0^* \sum_{t=0}^{\infty} \tilde{\beta}^t \left[\tilde{C}_t + b\tilde{L}_t\right] \tag{6}$$

subject to the budget constraint

$$\tilde{C}_t + Q_t(\tilde{L}_{t+1} - \tilde{L}_t) + \tilde{B}_{t+1} \leq R_{t-1}\tilde{B}_t, \qquad [\tilde{\lambda}_t] \tag{7}$$

where $\tilde{\lambda}$ denotes the Lagrange multiplier.

The first order conditions are

$$\begin{aligned}
C_t: &\quad \lambda_t = (C_t - \rho C_{t-1})^{-\rho} &(8)\\
L_{t+1}: &\quad Q_t\lambda_t = \beta E_t^*(\lambda_{t+1}Q_{t+1}) + \beta\gamma E_t^*(\lambda_{t+1}\frac{Y_{t+1}}{L_{t+1}}) + \theta\phi_t E_t^* Q_{t+1} &(9)\\
B_{t+1}: &\quad \lambda_t = \beta E_t^*(\lambda_{t+1}R_t) + \theta\phi_t R_t &(10)\\
\tilde{C}_t: &\quad \tilde{\lambda}_t = 1 &(11)\\
\tilde{L}_{t+1}: &\quad \tilde{\lambda}_t Q_t = \tilde{\beta} E_t^*(\tilde{\lambda}_{t+1}Q_{t+1}) + \tilde{\beta} b &(12)\\
\tilde{B}_{t+1}: &\quad \tilde{\lambda}_t = \tilde{\beta} E_t^*(\tilde{\lambda}_{t+1}R_t). &(13)
\end{aligned}$$

The market clearing condition for the good market is

$$C_t + \tilde{C}_t = A_t L_t^{\gamma}, \tag{14}$$

for the land it is

$$L_t + \tilde{L}_t = \bar{L}, \tag{15}$$

and for the asset market

$$B_t = \tilde{B}_t. \tag{16}$$

Coupled with an assumption on expectation formation process, equations (8)-(16) together with both budget constraints and the collateral constraint determine the solution of that model.

2.2 Rational expectations

Under rational expectations, $E_t^* = E_t$, both the interest rate R and the price of land Q are constant when $\theta = 1$. In that case this model has the same form as an RBC model with full depreciation, $\delta = 1$.

The linearized expectational system can be represented by the following three equations

$$\begin{aligned}
c_t &= \frac{1}{1+\rho}E_t^* c_{t+1} + \frac{\rho}{1+\rho}c_{t-1} - \frac{1}{\sigma}\frac{1-\rho}{1+\rho}\left[E_t^* a_{t+1} + (\gamma - 1)l_{t+1}\right] &(17)\\
l_t &= -\frac{1-\beta\gamma}{\beta\gamma}c_{t-1} + \gamma l_{t-1} + a_{t-1} &(18)\\
a_t &= \rho_a a_{t-1} + u_t, &(19)
\end{aligned}$$

where the lower-case letters denote the deviation from the steady state, $z_t = \frac{Z_t - Z}{Z}$.

We can write it in a vectorized form as

$$x_t = \mathbf{A} E_t x_{t+1} + \mathbf{B} x_{t-1} + \mathbf{u}_t, \tag{20}$$

where $x_t = [c_t,\ l_t,\ a_t]'$, $\mathbf{u}_t = [0,\ 0,\ u_t]'$ and matrices $\mathbf{A}$ and $\mathbf{B}$ are functions of underlying parameters.

The rational expectation solution to that system has a VAR(1) form

$$x_t = \mathbf{M}^{RE} x_{t-1} + \mathbf{u}_t, \tag{21}$$

where $\mathbf{M}^{RE}$, which is a function of structural parameters, solves the equation

$$\mathbf{M} = (\mathbb{I} - \mathbf{A}\mathbf{M})^{-1}\mathbf{B}. \tag{22}$$

2.3 Adaptive learning

Our key assumption is that households may not have rational expectations, i.e., $E_t^* \neq E_t$. Instead, we think of them as econometricians, who routinely estimate an equation that describes their view of the world, the perceived law of motion (PLM). We assume that agents' PLM is formulated in a way that is consistent with the RE solution (21) and they use this PLM to form their predictions about the future. In particular, the (time-varying) perceived law of motion that coincide with rational expectation equilibrium in equation (21) is

$$x_t = \mathbf{M}_t x_{t-1} + u_t, \tag{23}$$

but we allow $\mathbf{M}_t \neq \mathbf{M}^{RE}$, i.e., agents' perception may not match 1-to-1 to the rational expectations' solution.

As econometricians agents update their beliefs once new data becomes available. We assume that agents update their beliefs every period according to a special case of recursive least squares given by

$$\mathbf{M}_t = \mathbf{M}_{t-1} + \nu \mathbf{R}_t^{-1} x_{t-1}(x_t - \mathbf{M}_{t-1}' x_{t-1}) \tag{24}$$

$$\mathbf{R}_t = \mathbf{R}_{t-1} + \nu(x_{t-1}x_{t-1}' - \mathbf{R}_{t-1)}, \tag{25}$$

where ν is a small constant—households are assumed to use constant gain learning to update their parameter estimates.[4]

Agents use their most recent estimate of the perceived law of motion, M_t, to forecast the future state

$$E_t^* x_{t+1} = E_t\left(\mathbf{M}x_t + \mathbf{u}_t\right) = \mathbf{M}_{t-1}x_t \tag{26}$$

with $\mathbf{M}_t$ denoting the estimates obtained with the data up to date t. In the equation above we follow the following timing convention: at period t, when forming the expectations about $t+1$, agents do not observe x_t. Instead, they use their most recent forecast, i.e. $E_{t-1}x_{t+1} = M_{t-1}x_t$.[5]

Substituting these PLM-based expectations into equation (20) yield an actual law of motion (ALM) of that economy. In our case, putting equation (26) into equation (20) we obtain the actual law of motion under adaptive learning

$$x_t = \mathbf{A}\mathbf{M}_{t-1}x_t + \mathbf{B}x_{t-1} + u_t \Longrightarrow x_t = (\mathbb{I} - \mathbf{A}\mathbf{M}_{t-1})^{-1}\mathbf{B}x_{t-1} + (\mathbb{I} - \mathbf{A}\mathbf{M}_{t-1})^{-1}u_t \tag{27}$$

This equation combines structural parameters of the model (matrices $\mathbf{A}$ and $\mathbf{B}$) with the current beliefs (matrix $\mathbf{M}_{t-1}$) to govern the dynamics of x_t. Note that rational expectations equilibrium is a fixed point of that equation and the point, where the perceived and actual laws of motion of the economy coincide.

2.4 Simulations

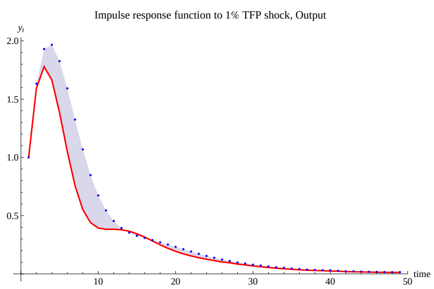
We use simulations to illustrate the importance of both initial beliefs and the confidence. This simple model is expectationally stable and a small departure of M_t from the rational expectations ($\mathbf{M}^{RE}$) will vanish over time. Nonetheless, the dynamics of the model given in (27) and the evolution of beliefs depend on the updating process (24)-(25). In particular, they depend on the initial belief matrix $\mathbf{M}_0$ and the variance-covariance matrix $\mathbf{R}_0$ that captures the confidence agents have with respect to their prior beliefs.

We set the value of leverage to $\theta = 1$, lender's discount factor as $\tilde{\beta} = 0.99$, borrower's discount factor to $\beta = 0.95 * 0.99$, the curvature of borrowers' utility function to $\sigma = 1$, and the land share $\gamma = 0.4$. The parameter measuring the importance of external habits in the utility function is set to $\rho = 0.9$. For the numerical simulations we employ constant gain learning with the gain parameter $\nu = 0.05$.

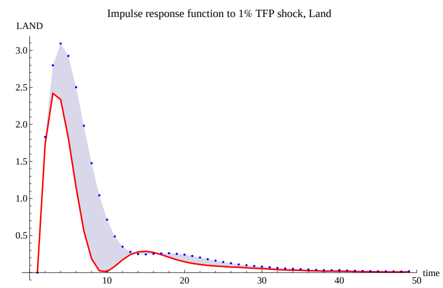
[4] Under recursive least squares, gain is a decreasing function of t with $\nu = \frac{1}{t}$.

[5] In our simulation this assumption does not make a significant difference for the IRFs or for the dynamics of $\mathbf{M}_t$.

Figure 1: Impulse response functions under learning (solid red) and RE (dotted blue) in case of the overestimation of $\mathbf{M_0}$



(a) Output response to 1% TFP shock.



(b) Borrower's land response to 1% TFP shock.

In the following subsections we look at impulse response functions to a one-time 1% productivity shock, for which the persistence is set at $\rho_a = 0.9$. For each parametrization we conduct 1000 simulations and report the median realization to characterize the typical dynamics. The precision of prior/initials beliefs, which can be thought of as confidence in priors, is represented by $\mathbf{R}_0^{-1}$, since the variance is equal to the inverse of precision. In particular, the smaller the $\mathbf{R}_0$ is the more diffused priors are the less confidence agents have in initial beliefs.

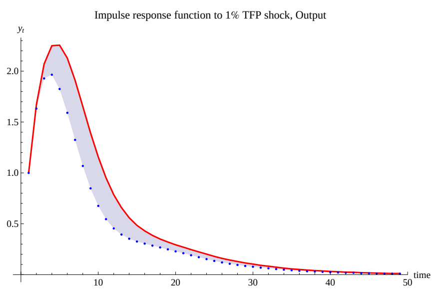
Initial beliefs not consistent with REE.

To illustrate the effects of initial beliefs and learning we allow agents' initial beliefs, $\mathbf{M}_0$, to be different from the rational expectations, that is $\mathbf{M}_0 \neq \mathbf{M}^{RE}$. In While we consider the case of $\mathbf{M}_0$ being 10% larger than $\mathbf{M}^{RE}$—in absolute terms, every entry of matrix $\mathbf{M}_0$ is 10% larger than the one in $\mathbf{M}_0$—we concentrate on the case in which we allow $\mathbf{M}_0$ to be 10% smaller. To set the precision matrix $\mathbf{R}_0$ we simulate the rational expectation model (calibrated using parameter values presented above) using $\mathbf{M}^{RE}$ and take the estimate of the variance-covariance matrix $\hat{\mathbf{R}}$ as initial $\mathbf{R}_0$.

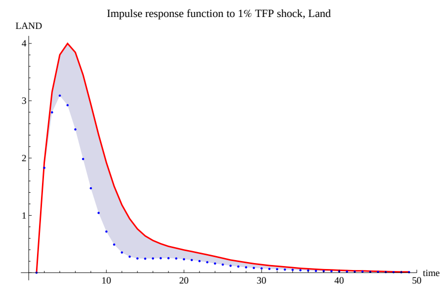
Figure 1 depicts the impulse responses of the output and the stock of land held by borrowers to the 1% productivity shock for the case in which agents overestimate (relative to RE) VAR(1) matrix.

Figure 2 depicts the analogous impulse response functions for the case in which agents underestimate (relative to RE) VAR(1) matrix.

Figure 2: Impulse response functions under learning (solid red) and RE (dotted blue) in case of the underestimation of $\mathbf{M_0}$



(a) Response of output to 1% TFP shock.

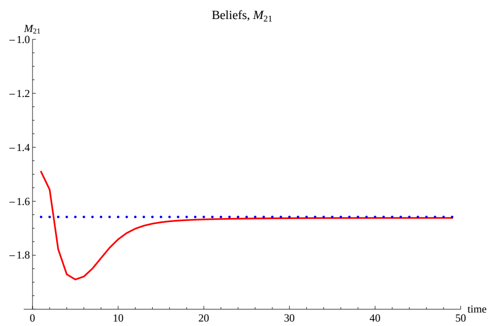


(b) Response of borrower's to 1% TFP shock.

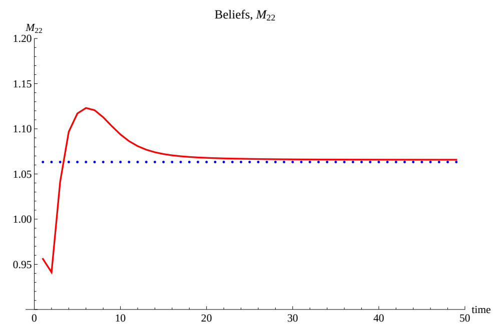
In case of the underestimation of matrix $\mathbf{M}_0$, the output, the borrowers' consumption and the borrowers' land respond more to a 1% productivity shock under learning than under rational expectations. This difference, however, decreases over time as agents learn and matrix $\mathbf{M}_t$ converge to $\mathbf{M}^{RE}$. To illustrate this convergence of beliefs we plot the evolution of $\mathbf{M}_t$. Since $\mathbf{M}$ is 3×3 we plot the entries that illustrate the behavior of land and output (since $y_t = a_t l_t^{\gamma}$.) Figure 3 shows entries that corresponds to the reaction of the borrower's land to the last period borrowers' consumption, the previous period stock of land, as well as the productivity, respectively. The initially larger responses of the land and the output under learning originate from the reduced reaction of land to the higher consumption (entry $M_{21,t}$) as well as the underestimation of the persistence of the shock (entry $M_{33,t}$ depicted in the bottom right panel of Figure 3). However, over time agents learn and eventually matrix $\mathbf{M}_t$ becomes arbitrarily close to the one corresponding to the rational expectations. The speed of this process is not uniform even though all elements of matrix $\mathbf{M}_t$ converge to $\mathbf{M}^{RE}$. It is worth to point out that agents overreact in their revisions: with the subsequent realizations of endogenous variables, which are govern by the actual law of motion, agents recognize that they underestimated the size/coefficients of matrix $\mathbf{M}$. They, however, over-correct their beliefs relative to the rational expectations — while they were initially underestimating the size of $\mathbf{M}$, after the 2-3 periods they tend to overestimate its magnitude. This is due the revision process governed by both the impact of the forecast errors in equations (24) and (25) and the actual dynamics governed by the ALM ((27) in the presence of the relatively low confidence, represented by matrix $\mathbf{R}_t$.[6]

[6]This pattern does not rely on agents having wrong beliefs about the persistence of the stochastic process. Figure 14 in the Appendix A presents the results for which agents' initial beliefs with respect to ρ are centered around the true value but all other elements of matrix $\mathbf{M}_0$ are underestimated.

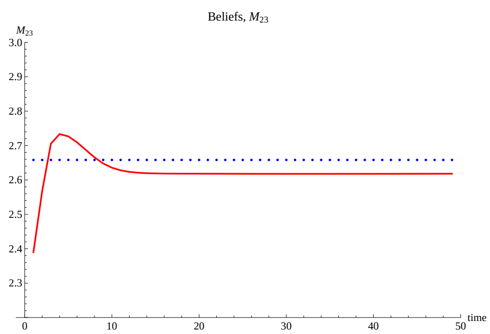
Figure 3: Evolution of beliefs under learning (solid red) and RE (dotted blue) in case of the underestimation of $\mathbf{M_0}$



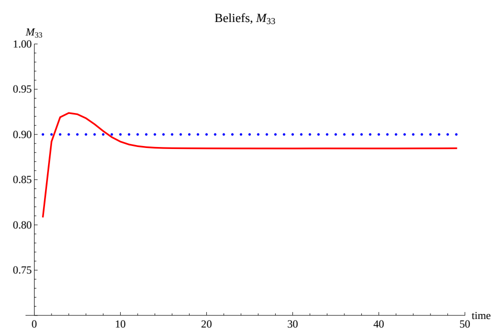
(a) Evolution of $M_{21,t}$



(b) Evolution of $M_{22,t}$

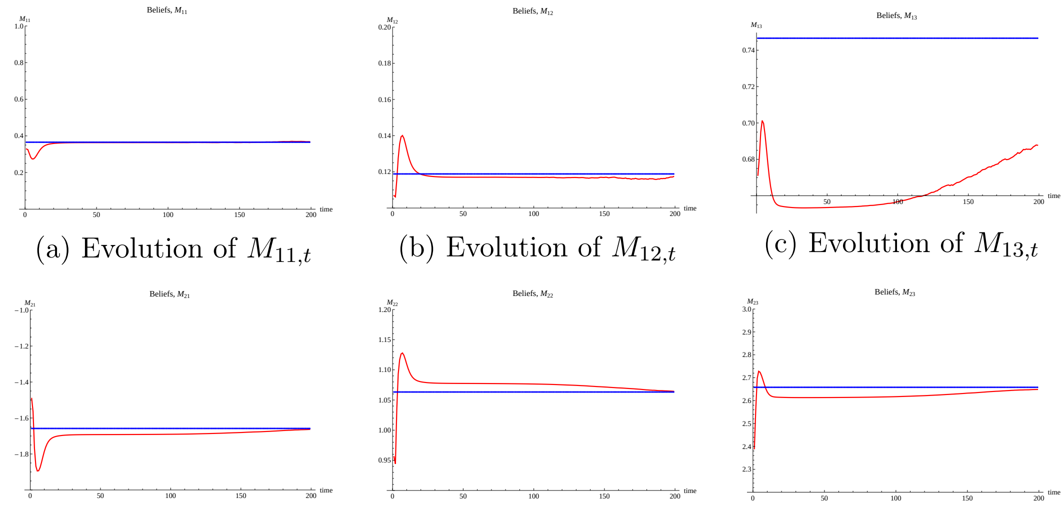


(c) Evolution of $M_{23,t}$



(d) Evolution of $M_{33,t}$

Figure 4: Convergence of beliefs under learning (solid red) and RE (dotted blue) in case of the underestimation of $\mathbf{M_0}$.



(a) Evolution of $M_{11,t}$ (b) Evolution of $M_{12,t}$ (c) Evolution of $M_{13,t}$

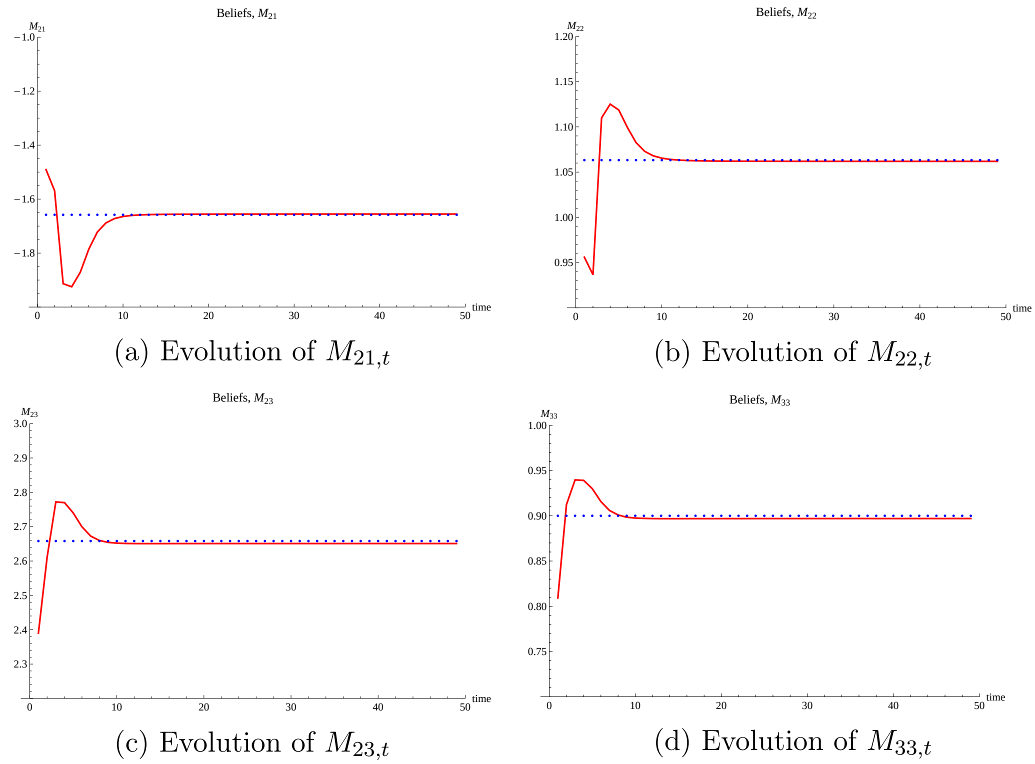
(d) Evolution of $M_{21,t}$ (e) Evolution of $M_{22,t}$ (f) Evolution of $M_{23,t}$

Figure 4 illustrates that over long enough time the beliefs become consistent with rational expectations.

Precision of the initial beliefs

We now present the extent to which the confidence, measured by the precision matrix, in initial beliefs matters for both the dynamics of endogenous variables and the evolution of beliefs. Consider the case in which the prior beliefs are more diffused/dispersed—by making the matrix $\mathbf{R}_0$ smaller we reduce the initial beliefs' precision and increase the variance of $\mathbf{M}_0$. Such change speeds up the process of updating the beliefs. Equation (24) shows how $\mathbf{R}_0$ affects the evolution of the matrix $\mathbf{M}_t$. Smaller $\mathbf{R}_0$ implies larger $\mathbf{R}_0^{-1}$ and yields bigger impact of current forecast errors, $(x_t - \mathbf{M}'_{t-1}x_{t-1})$, on the new estimate of the coefficients. The less confident agents are in their initial beliefs the bigger the revisions of their beliefs due to observed forecast error. Figure 5 depicts the evolution of the matrix $\mathbf{M}$ for the case of $R_0 = 0.1 \times R^{RE}$ with initially underestimated matrix $\mathbf{M}_0$. The convergence of beliefs to the REE occurs here within 7 periods whereas in the case previous case it lasted almost twice as long. Fast convergence of beliefs does not imply, however, that the path under learning of endogenous variables start to match at the same the impulse responses under rational expectations. Figure 6 illustrates how dispersed beliefs affect the evolution of the output and the land holdings of borrowers—despite the rapid convergence of $\mathbf{M}_t$ to $\mathbf{M}^{RE}$ the initial departure of beliefs from rational expectation has lingering effects on the output and the stock of land held by borrowers.

Figure 5: Evolution of beliefs under learning (solid red) and RE (dotted blue) in case of the underestimation of $\mathbf{M_0}$ for dispersed beliefs.



(a) Evolution of $M_{21,t}$

(b) Evolution of $M_{22,t}$

(c) Evolution of $M_{23,t}$

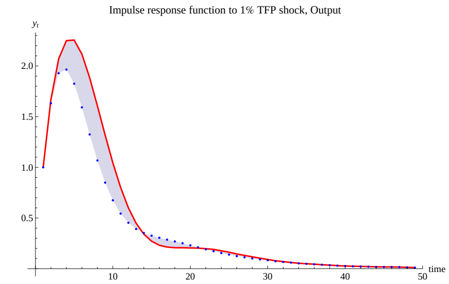
(d) Evolution of $M_{33,t}$

This pattern of response is symmetric: if agents' overestimate matrix $\mathbf{M}$ the results are reversed with initially muted response of output and borrowers' land to a productivity shock only to be growing over time. Figure 15 in the appendix presents impulse response functions to a 1% shock with the matrix $\mathbf{M}$ initially 10% larger than $\mathbf{M}^{RE}$ and $\mathbf{R}_0 = 0.1 \times \mathbf{R}^{RE}$.

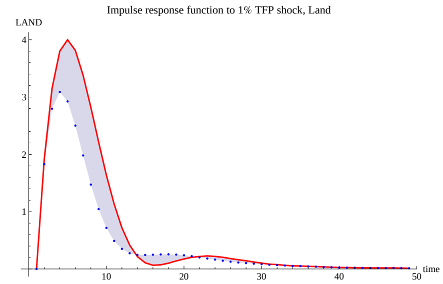
What if agents had even less confidence in their initial beliefs? Figures 7 and 8 depict both the IRFs and the evolution of beliefs $\mathbf{M}_t$. While there is a clear convergence to the rational expectations, the dynamic response of the entire system is very perturbed. The path of $\mathbf{M}_t$ moves faster towards $\mathbf{M}^{RE}$ due to even less confidence in initial beliefs but the rapid changes of beliefs cause the endogenous variables to fluctuate with oscillatory trajectories around the RE path. Such behavior is driven by the changes of the agents' perception: the revisions of beliefs coupled with forecasts errors—due to both the shock itself and the incorrect perception of how the system behaves—induce agents to overreact in their response to the shock.

The reverse is true if agents have more confidence in their initial beliefs. In that

Figure 6: Impulse response functions under learning (solid red) and RE (solid blue) in case of the underestimation of $\mathbf{M_0}$ under dispersed initial beliefs.

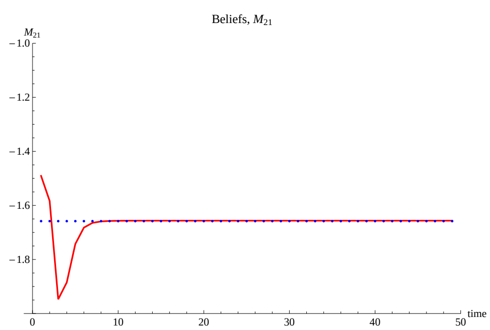


(a) Response of Output to a 1% TFP shock.

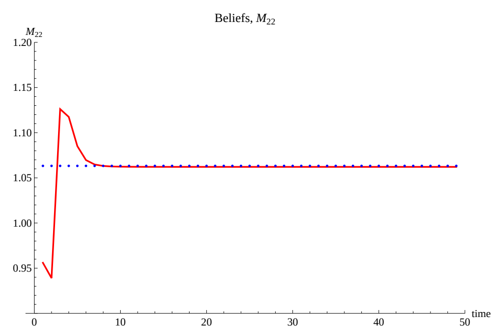


(b) Response of borrower's land to a 1% TFP shock.

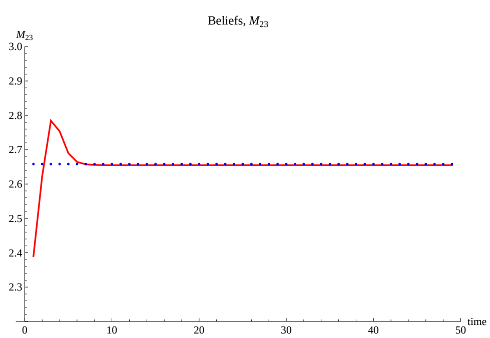
Figure 7: Evolution of beliefs under learning (solid red) and RE (dotted blue) in case of the underestimation of $\mathbf{M_0}$ under very dispersed initial beliefs.



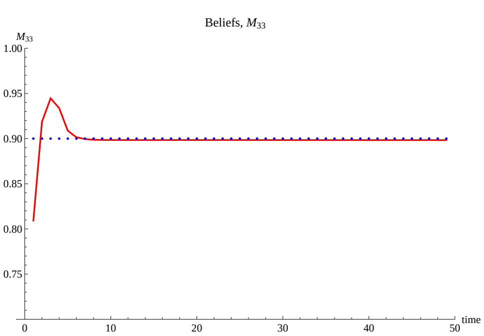
(a) Evolution of $M_{21,t}$



(b) Evolution of $M_{22,t}$

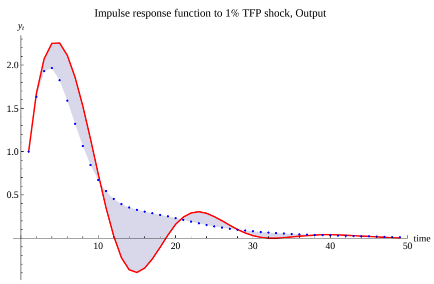


(c) Evolution of $M_{23,t}$

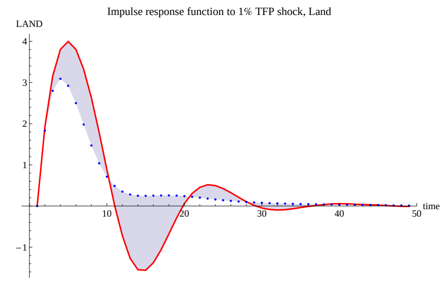


(d) Evolution of $M_{33,t}$

Figure 8: Impulse response functions under learning (solid red) and RE (solid blue) in case of the underestimation of $\mathbf{M_0}$ under very dispersed initial beliefs.

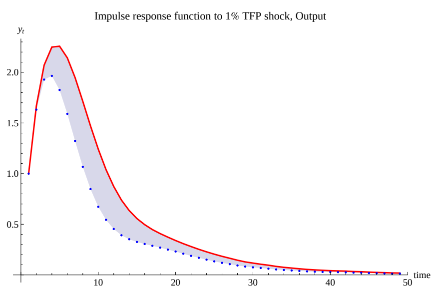


(a) Output response to 1% TFP shock.

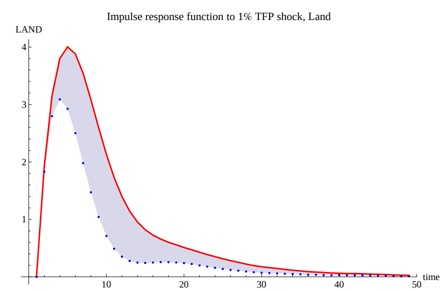


(b) Borrower's land response to 1% TFP shock.

Figure 9: Impulse response functions under learning (solid red) and RE (solid blue) in case of the underestimation of $\mathbf{M_0}$ under strong initial beliefs.



(a) Output response to 1% TFP shock.



(b) Borrower's land response to 1% TFP shock.

case the convergence of beliefs and the responses of endogenous variables can be very slow. Figure 10 presents the evolution of matrix $\mathbf{M}_t$ and Figure 9 shows the behavior of endogenous variables if agents are relatively confident in their initial beliefs.[7] Since the agents are more confident in their priors, they do not revise their beliefs as quickly as in the cases analyzed so far. This affects the dynamic response of the system, which now remains persistently away from the one governed by the rational expectations.

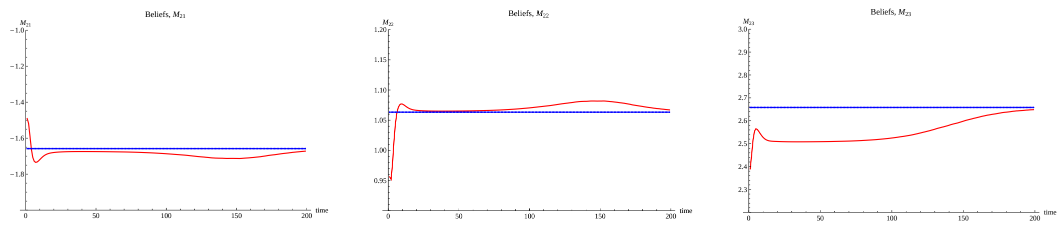
Summary

Both the perception of the economy's law of motion and the degree of the conviction that agents have are determining the dynamics of the entire system.[8] Even in an expectationally stable system, in which agents' PLM eventually coincide with ALM,

[7]The initial variance-covariance matrix is 10 times larger than the one associated with OLS estimates of R.

[8]This result extends to large models: Pintus, Suda and Turgut (2021) show that confidence matter for the perception of the economy's law of motion also in a medium size estimated model.

Figure 10: Evolution of beliefs under learning (solid red) and RE (dotted blue) in case of the underestimation of $\mathbf{M_0}$ under strong initial beliefs.



(a) Evolution of $M_{21,t}$ (b) Evolution of $M_{22,t}$ (c) Evolution of $M_{23,t}$

the dynamics of the learning system along the convergence path can be substantially different from the rational expectations dynamics.

This result is particularly important from the policy design and policy implementation perspectives. Lucas (1976) critique stresses that expectations and the reaction to policy changes are endogenous and should not be treated as constant. This section illustrates how, in the very simple model with adaptive learning, prior beliefs affect expectations, their evolution, and the path of entire economy. In the next two sections we show how initial beliefs and confidence can affect the policy implementation.

3 Endogenous collateral constraint model

In this section we extend the basic model from section 2 and introduce the capital, the endogenous labor and the endogenous stochastic leverage while retaining the adaptive learning assumption.

Consider a small open economy with a representative agent that has the following maximization problem

$$\max_{C_t,N_t,K_{t+1},L_{t+1},N_t,B_{t+1}} E_0^* \sum_{t=0}^{\infty} \beta^t \frac{\left[C_t - \psi \frac{N_t^{1+\chi}}{1+\chi}\right]^{1-\sigma} - 1}{1-\sigma}, \tag{28}$$

where C_t denotes the consumption, N_t is the hours worked, σ denotes the relative risk aversion, and χ measures the elasticity of the labor supply. The maximization is subject to the budget constraint:

$$C_t + K_{t+1} - (1-\delta)K_t + Q_t(L_{t+1} - L_t) + (1+R)B_t = B_{t+1} + AK_t^{\alpha} L_t^{\gamma} N_t^{1-\alpha-\gamma} \tag{29}$$

and the collateral constraint:

$$\tilde{\Theta}_t E_t^*[Q_{t+1}]L_{t+1} \geq (1+R)B_{t+1}, \tag{30}$$

where K_{t+1} is the capital stock, L_{t+1} represents the land stock, B_{t+1} denotes the amount of new borrowing, Q_t is the land price, and A is (constant) total factor productivity. Given the small open economy assumption, the real interest rate, R, is exogenous and assumed to be constant. Importantly, $\tilde{\Theta}_t$ denotes endogenous leverage that responds to both changes in the land price and the exogenous shocks:

$$\tilde{\Theta}_t \equiv \Theta_t \left\{ \frac{E_t[Q_{t+1}]}{Q} \right\}^{\varepsilon}, \tag{31}$$

where Q is the steady-state value of the land price and the log of Θ_t ($\theta_t = \log(\Theta_t)$ follows an AR(1) process,

$$\theta_t = (1-\rho_\theta)\overline{\theta} + \rho_\theta \theta_{t-1} + \xi_t, \tag{32}$$

where ξ_t is the leverage shock. The parameter ε captures the extent to which asset prices affect the leverage. Using the US micro data Mian and Sufi (2011) show that in the period preceding the crisis the leverage was pro-cyclical in house prices.

The first-order conditions of maximizing (28) subject to the budget constraint (29) and the collateral constraint (30) are as follows

$$\left[C_t - \psi \frac{N_t^{1+\chi}}{1+\chi} \right]^{-\sigma} = \Lambda_t \tag{33}$$

$$\psi N_t^{\chi+\alpha+\gamma} = (1-\alpha-\gamma)AK_t^{\alpha}L_t^{\gamma} \tag{34}$$

$$T_t Q_t \Lambda_t = \beta E_t[T_{t+1}Q_{t+1}\Lambda_{t+1}] + \beta\gamma E_t[\Lambda_{t+1}Y_{t+1}/L_{t+1}] + \Phi_t \tilde{\Theta}_t E_t[Q_{t+1}] \tag{35}$$

$$\Lambda_t = \beta E_t[\Lambda_{t+1}(\alpha Y_{t+1}/K_{t+1} + 1 - \delta)] \tag{36}$$

$$\Lambda_t = \beta(1+R)E_t[\Lambda_{t+1}] + (1+R)\Phi_t, \tag{37}$$

where Λ_t and Φ_t denote the Lagrange multipliers of constraints (29) and (30), respectively.

Taking the first-order conditions and linearizing them around the steady state allows us to write down a linear expectational system

$$X_t = \mathbf{N}_t + \mathbf{A}X_{t-1} + \mathbf{B}E^*_{t-1}X_t + \mathbf{C}E^*_t X_{t+1} + \mathbf{D}\xi_t \ , \tag{38}$$

where lower case letters denote variables in log, $X_t = \{c_t, q_t, b_t, k_t, l_t, \lambda_t, \phi_t, \theta_t\}'$ is the vector of endogenous variables; matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ and $\mathbf{N}$ are functions of structural parameters; $\xi_t \equiv \log \Xi_t$ is the exogenous leverage shock and, as in Section 2, E^*_t denotes the potentially non-rational expectations.

Under rational expectations $E^*_t = E_t$ and for $1 + R < \frac{1}{\beta}$ there exists a unique rational expectation solution with a binding collateral constraint (30). The minimal state variable (MSV) solution to that system is

$$X_t = \mathbf{H}^{RE} + \mathbf{M}^{RE}X_{t-1} + \mathbf{G}^{RE}\xi_t \, , \tag{39}$$

where $\mathbf{M}^{RE}$ is the solution to

$$\mathbf{M} = [\mathbb{I} - \mathbf{C}\mathbf{M}]^{-1}[\mathbf{A} + \mathbf{B}\mathbf{M}] \tag{40}$$

and $\mathbf{H}^{RE}$ and $\mathbf{G}^{RE}$ are given by

$$\mathbf{H}^{RE} = [\mathbb{I} - \mathbf{C}(\mathbf{M}^{RE} - \mathbb{I}) + \mathbf{B}]^{-1}\mathbf{N} \tag{41}$$

$$\mathbf{G}^{RE} = \left[\mathbb{I} - \mathbf{C}\mathbf{M}^{RE}\right]^{-1}\mathbf{D} \ . \tag{42}$$

The rational expectations hypothesis implies that both perceived and actual law of motions are described by (39): agents perceived probability distributions of endogenous and exogenous variables agrees with the true distributions governing the system's dynamics. In this setting, a policy change—whether it is a fiscal policy and an introduction of a lump-sum or distortionary taxes or a macro-prudential policy taking form of financial regulations constraining the leverage—alters one (or more) of matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$, and/or $\mathbf{N}$. This change is reflected, in turn, in the equilibrium described by matrices $\mathbf{M}^{RE}$, $\mathbf{H}^{RE}$, and/or $\mathbf{G}^{RE}$. The rational expectations implicitly assume that households understand that and their forecasts following the policy change are consistent with these new matrices.

We relax the rational expectation assumption and replace it with the adaptive learning approach described in section 2. Agents use the perceived law of motion

$$X_t = \mathbf{H} + \mathbf{M}X_{t-1} + \mathbf{G}\xi_t \tag{43}$$

for forecasting. The key element that differentiates equation (43) from its rational expectations' counterpart in equation (39) is that we do not assume that $\mathbf{H} = \mathbf{H}^{RE}$, $\mathbf{M} = \mathbf{M}^{RE}$, $\mathbf{G} = \mathbf{G}^{RE}$. Given that $E_t X_{t+1} = \mathbf{H}_{t-1} + \mathbf{M}_{t-1} X_t$ and $E_{t-1} X_t = \mathbf{H}_{t-2} + \mathbf{M}_{t-2} X_{t-1}$ the actual law of motion is given by

$$[\mathbb{I} - \mathbf{C}\mathbf{M}_{t-1}]\, X_t = [\mathbf{A} + \mathbf{B}\mathbf{M}_{t-2}]\, X_{t-1} + [\mathbf{B}\mathbf{H}_{t-2} + \mathbf{C}\mathbf{H}_{t-1} + \mathbf{N}] + \mathbf{D}\xi_t. \tag{44}$$

As in section 2 we assume that agents update their beliefs every period using the recursive constant gain algorithm:

$$\beta_t = \beta_{t-1} + \nu \mathbf{R}_t^{-1} Z_{t-1}(Z_t - \beta_{t-1}' Z_{t-1}) \tag{45}$$

$$\mathbf{R}_t = \mathbf{R}_{t-1} + \nu(Z_{t-1} Z_{t-1}' - \mathbf{R}_{t-1)}, \tag{46}$$

where $\beta_t = [\mathbf{H}_t \quad \mathbf{M}_t]$ is a time-varying matrix of coefficients, $Z_t = [1 \quad X_t']'$ and $\mathbf{R}_t$ is the associated variance-covariance matrix. Equation (45) describes the path of beliefs (represented by β_t) given some initial beliefs β_0. Similarly, equation (46) presents the evolution of variance-covariance of estimates $\mathbf{M}_t$ given the confidence in priors beliefs, $\mathbf{R}_0$.

Under rational expectations the dynamics of all endogenous variables are determined entirely by the equation (39). The dynamics under adaptive learning are, in turn, jointly determined by equations (44)-(46) and conditional on β_0 and $\mathbf{R}_0$.

Having set the stage we can now analyze the effect of a new macroeconomic (macro-prudential) policy that is trying to reduce the volatility induced by the financial shocks.

4 The dangers of macro-prudential policy experiments

In this section we show that the introduction of macro-prudential policy that under rational expectations reduces considerably the volatility of endogenous variables yield the opposite under learning. To make this result explicit we compare the economy's impulse responses to a financial shock before and after the introduction of such policy.

For the numerical exercise we follow the calibration of Pintus and Suda (2019), see Table 1. Such calibration delivers average values for the leverage ($\bar{\Theta} \approx 0.88$), debt-to-GDP ($\frac{B}{Y} \approx 0.52$) and land value-to-GDP ($\frac{QL}{Y} \approx 0.59$) ratios observed in the period 1996Q1-2008Q4, that is preceding the Global Financial Crisis. Setting $\mu = 0.99$ to reflect the annual real interest rate of 4% , the time preference parameter to $\beta = 0.96\mu$, the inverse of labor elasticity to $\chi = 1/3$ the capital share $\alpha = 0.33$, and land share

Parameter	Value	Source/Target
μ	0.99	$R = 4\%$
β	0.96μ	Iacoviello (2005)
δ	0.025	
α	0.33	Gertler et al. (2012)
γ	0.0093	$\frac{QL}{Y} \approx 0.59$
$\bar{\Theta}$	0.88	Pintus and Suda (2019)
χ	$\frac{1}{3}$	Gertler et al. (2012)
ν	0.014	Pintus et al. (2021)
σ_ξ	0.033	Pintus et al. (2021)

Table 1: Parameter values

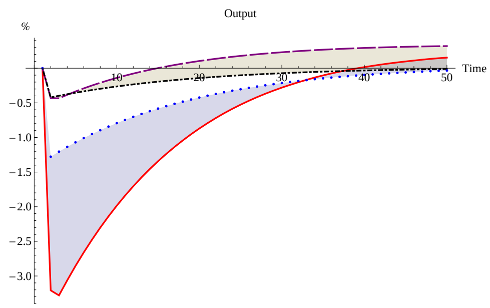
$\gamma = 0.0093$ deliver these ratios. The key parameter in our exercise is the land price elasticity of leverage ε. Using the data on 2002-2006 changes in house prices and debt-to-income, Mian and Sufi (2011) find evidence of mildly pro-cyclical (in housing prices) leverage. We set $\varepsilon = 0.5$ to match their results on the impact of housing price changes on the debt-to-income ratio.

Pintus and Suda (2019) show that learning can significantly amplify leverage shocks when agents' beliefs about the model parameters differ from the rational expectations. In particular, they show that if households overestimate the persistence of the financial shock process and the leverage is in fact mildly pro-cyclical (represented by $\varepsilon = 0.5$) the financial shock ξ_t causes over 2.5 times larger response of output, capital, and consumption under learning than under rational expectations. Not only adaptive learning amplifies economic shocks, but also the actual effect is quantitatively large: a large negative shock to leverage of about -5% reduces the output by around 3.2% under learning but only by about 1,3% under rational expectations.

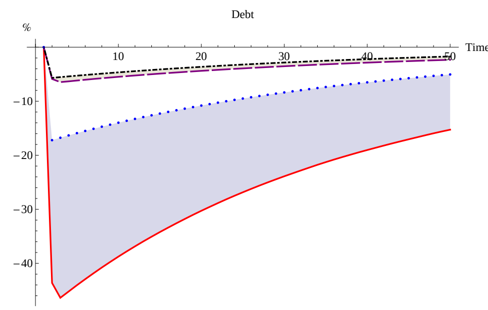
The foundation of the economy's large response to a leverage shock under learning lies in the interaction of the forecast of land prices with the borrowing constraint in equation (30). If a negative shock to the leverage would not translate into the fall in land prices and, in turn, would not lower the value of collateral resulting in less borrowing, the learning economy would behave like a RE economy. Moreover, eliminating the effect of land price swings on the borrowing constraint would not only bring the dynamics under learning closer to the ones under RE but would also reduce the response under rational expectations.

Consider now a macro-prudential policy that makes the leverage countercyclical in

Figure 11: Responses under pro-cyclical ($\varepsilon = 0.5\%$) leverage for learning (solid red) and RE (dotted blue) and counter-cyclical ($\varepsilon = -0.75\%$) leverage for learning (dashed purple) and RE (dashed-dotted black) in case of the overestimation of the persistence of leverage shocks,ρ .



(a) Output following 5% leverage shock.



(b) Debt following 5% leverage shock.

housing/land prices.[9] If such policy implies $\varepsilon = -0.75$ and adaptive learning agents have a correct understanding how this new ε affects $\mathbf{M}_{\tau}$, the economy's dynamic responses under learning and RE are greatly reduced and the path of the economy with the adaptive learning is considerably closer to the one under rational expectations. Figure 11 depicts the impulse responses under adaptive learning and rational expectations before and after the introduction of such a policy. Similarly to Pintus and Suda (2019) the learning amplifies the effects of leverage shocks on output and debt by a factor of 2.5 for the pro-cyclical leverage but the difference almost disappears if the leverage is counter-cyclical. The introduction of the macro-prudential policy reduces the debt response to the leverage shock from -35% to -6% under learning and from -16% to -6% under RE. This result seems to provide unequivocal support for the macro-prudential policy if one wants to reduce the fluctuations following the financial shocks, especially in the case of imperfect information and learning.

4.1 Surprising policy experiment

However, this impressive policy success does not materialize if households are not aware of the effect of that new macro-prudential policy on the cyclicality of the collateral constraint. This would be the case if policy makers experiment in a discretionary fashion. Consider the case that the policy makers introduce the procyclical taxes that would result, under rational expectations, in the counter-cyclical leverage but assume that

[9]Such a policy could be introduced via procyclical taxes in a model of moral hazard and costly monitoring.

learning agents have initially no information about that because the policy experiment is not advertised or announced. In terms of our notation, agents' PLM does not reflect negative ε but the positive one from before the introduction of that policy. Figure 12 presents impulse responses in such case. In this figure households do not overestimate the persistence of the financial shock.

Following the negative leverage shock, we observe a considerable larger fall in debt under learning than under rational expectations, but this negative effect drops considerably faster than under the case of pro-cyclical leverage. However, under adaptive learning but with incorrect beliefs the response of output to the financial shock is significantly different. Following the immediate drop in output (yet it is only half of magnitude of the one in Figure 11) the economy is booming in just 12 periods following the shock—all despite negative leverage shock. If the purpose of that policy was to decrease the financial shock volatility of endogenous variables, then the implementation of countercyclical policy fails miserably.

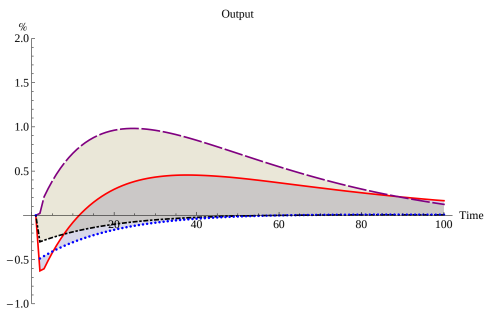
4.2 Confidence in beliefs

To draw the impulse responses functions in Figure 12 we assume that at the moment of the deployment of the countercyclical macro-prudential policy households are completely oblivious to that change. From the perspective of the model, this implies that at that very moment not only matrix $\mathbf{M}_{policy\ change}$ corresponds to the case RE matrix $\mathbf{M}^{RE}_{\varepsilon>0}$ with pro-cyclical leverage ($\varepsilon > 0$) but also the variance-covariance matrix $\mathbf{R}_{policy\ change}$ correspond to the RE dynamics.

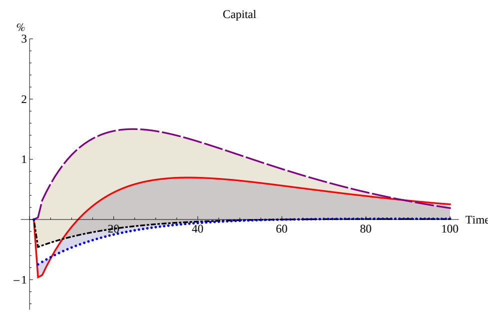
Would some form of policy announcements that decreases the households' confidence in $\mathbf{M}_{policy\ change}$ reduce the boom or speed up the convergence to new $\mathbf{M}^{RE}_{\varepsilon<0}$? Figure 13 depicts the case in which $\mathbf{M}_{policy\ change} = \mathbf{M}^{RE}_{\varepsilon>0}$ but households are uncertain about the new relationships between land prices and debt and the rest of the economy. If the policymaker manages to announce its new policy in a way that makes agents more open and more likely to update their beliefs with the new data (less confidence approximated by larger variance) the dynamics can be even more surprising with even larger economic boom and the increase of debt. It is clear from Figures 12 and that the introduction of new macro-prudential regulations or conducting the announced policy experiments without properly addressing how this affect the economy and, therefore, guiding agents' beliefs, can result in significantly higher volatility than without such change.

To quantify the increase in volatility we calculate the standard deviation of endogenous variables for a number of simulations. We find that the introduction of macro-

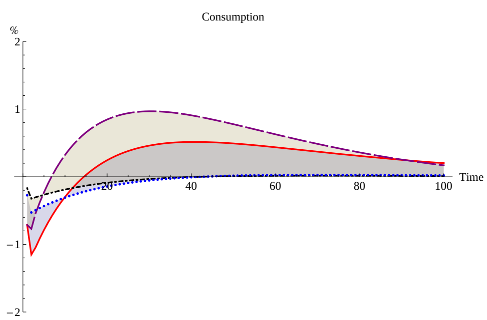
Figure 12: Responses to a negative leverage shocks for mildly counter-cyclical ($\varepsilon = -0.5\%$) leverage under learning (solid red) and RE (dotted blue) and strongly counter-cyclical ($\varepsilon = -1.5\%$) leverage under learning (dashed purple) and RE (dashed-dotted black) given the incorrect beliefs regarding the macro-prudential policy.



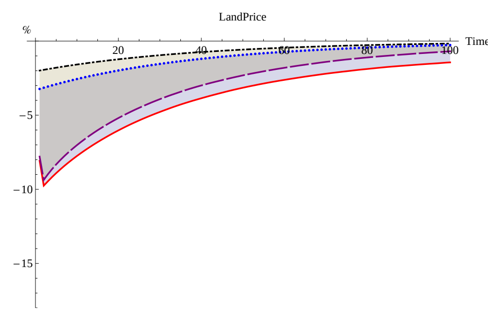
(a) Output following 5% leverage shock.



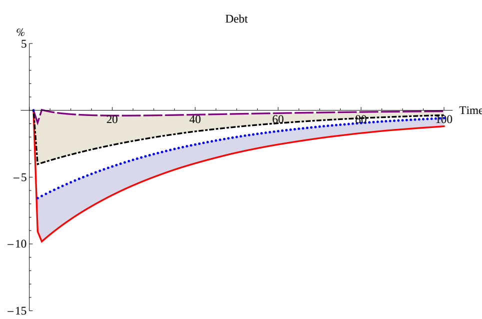
(b) Capital following 5% leverage shock.



(c) Consumption following 5% leverage shock.

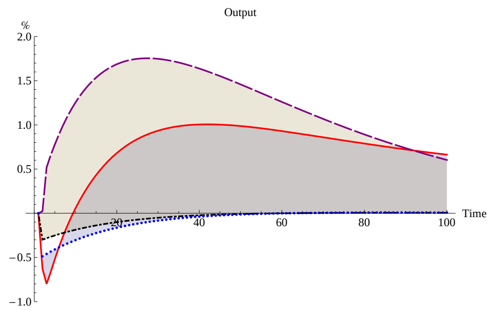


(d) Land price following 5% leverage shock.

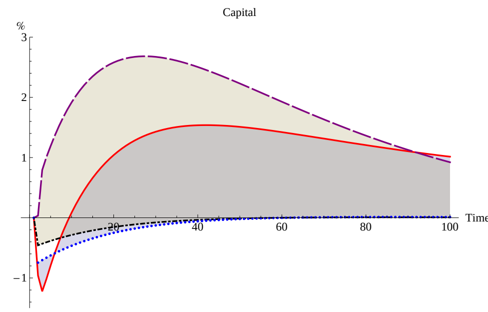


(e) Debt following 5% leverage shock.

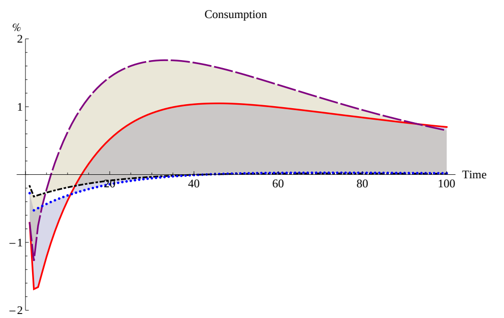
Figure 13: Responses to a negative leverage shocks for mildly counter-cyclical ($\varepsilon = -0.5\%$) leverage under learning (solid red) and RE (dotted blue) and strongly counter-cyclical ($\varepsilon = -1.5\%$) leverage under learning (dashed purple) and RE (dashed-dotted black) given the incorrect beliefs regarding the macro-prudential policy but with less confidence.



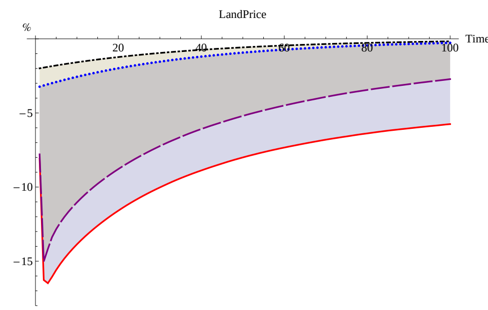
(a) Output following 5% leverage shock.



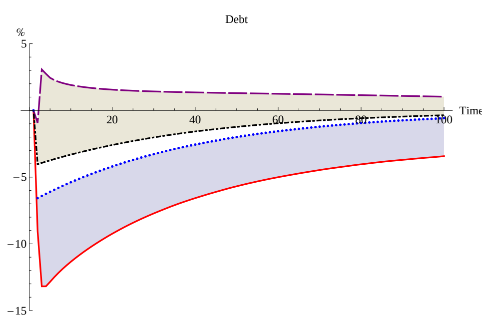
(b) Capital following 5% leverage shock.



(c) Consumption following 5% leverage shock.



(d) Land price following 5% leverage shock.



(e) Debt following 5% leverage shock.

prudential policy increases the standard deviation of consumption increases by a factor of 2 with even higher increase for the land prices.

The proper accounting for expectations and prior beliefs is paramount for the successful implementation of any policy.

5 The implication for the design and introduction of new macroeconomic policy

Just like in our simple model the evolution of beliefs and hence the dynamics of the endogenous variables do depend on the confidence agents have in their initial beliefs. The higher the confidence and the lower the variance of priors, the less revision observed in the data. If, on the other hand, agents are not very certain about the particular values in matrix $\mathbf{M}$ the revision of beliefs can be significant.

This result has potentially very big impact for the policy design. Any change of economic policy, any introduction of the a parameter or an element that changes the existing PLM can be associated with agents having to determine their perception of this policy. If at the time of introduction of that policy households and firms have no data to base their beliefs on, it should be the goal for the policymakers to convince these agents how this policy will operate. Moreover, strict adherence to a newly introduced policy is vital to confirm the perception and expectations of agents on the effects of this policy on economic outcome.

Consider the case of a central bank announcing that it will take into account additional information / dynamics when setting the nominal interest rate. Only if the central bank stick to this policy will agents be able to positively verify the policy announcement and confirm the message. If, however, the policy is not followed either due to policymaker choice or lack of possibility, the perception of agents on that policy will be revised.

This might be particularly important for the case where the policymaker announces new policy but is unable to implement it. Imagine that the central bank promises to keep the interest rate low even if the commonly used Taylor rule predicts raising them. Until such thing actually occurs, households and firms are unable to update their beliefs on such policy. Ultimately the policymakers need not only talk the talk but then walk the walk to be able to successfully implement new policy.

The story we built in a small model is confirmed in much larger and more qualitatively and quantitatively accurate model in section 3. Figures 12 and 13 bear an important implication for any new macroeconomic policy. Households' and firms' choices are based on both current and expected future economic condition that affects agents' objectives and constraints. Once we relaxed the assumption of rational expectations and replaced it with adaptive learning approach, these expectations are based on subjective probability distribution and perceived law of motion, both of which may not be the same as actual law of motion and actual probability distributions (of both endogenous and exogenous variables). Moreover, given that agents update their PLM, their forecasts evolve over time not only due to changing environment but also due to updated perception of the linkages within that environment.

This makes an implementation of any policy contingent on agents' reckoning of this policy and how it evolves. In some cases, these do not pose any difficulty due to the nature of the policy. For example, the announced and enacted change of tax rate is likely to cause very little disturbance in the expectations as far as the PLM is concerned: agents have already learnt how the taxes work and the variation in their rate is not likely to be associated with big uncertainty on how it affects individual and aggregate constraint. If, however, the completely new policy is introduced—be it new unconventional monetary policy measure, macroprudential policy or even the change of the Taylor rule—there is likely to be much more uncertainty on i) how such new policy works, ii) through which channels it is likely to operate and ultimately affect the economy, and iii) how it is implemented. All these elements are important for expectation formation process. Our results in sections 2 indicate that both agents' perception and associated with this perception uncertainty affect how an economy responds to these policies at the moment of their announcement, introduction and, even more importantly, following their implementation. At the same time, the results on the failure of macro-prudential policy presented in section 4 show that only properly accounting for beliefs can yield desired results.

6 Conclusion

In the world in which people do not know everything but observe and learn, the initial perception can be make it or break it for the economy. Using calibrated models, we show that the degree of confidence that households have in their perception of the law of motion have large impact on how this perception changes and on economy itself. We then show that the deployment of even the best policies can be very costly.

The Lucas critique put rational expectations into the forefront of macroeconomics and revolutionized how economists were thinking about expectations and economic policy. It leads to important results from rules versus discretion to policy-dependent determinacy of equilibria to the importance of public vs private signals to many other important lessons for the policy design and the policy implementations.

We should not forget, however, that even temporary deviations from rational expectations can change what one can consider as good or desirable policy. Designing a macroeconomic policy that is robust to such deviations could prove difficult but rewarding in the quest for the optimal policy.

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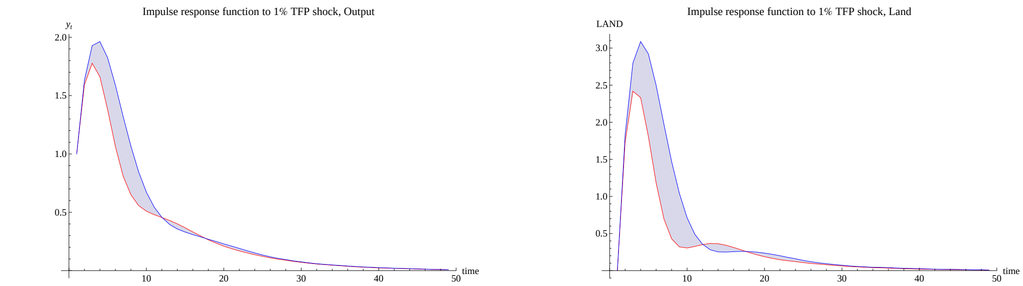
Figure 15: Responses under learning (solid red) and RE (solid blue) in case of the overestimation of $\mathbf{M_0}$ under dispersed beliefs.



(a) Output following 1% productivity shock.

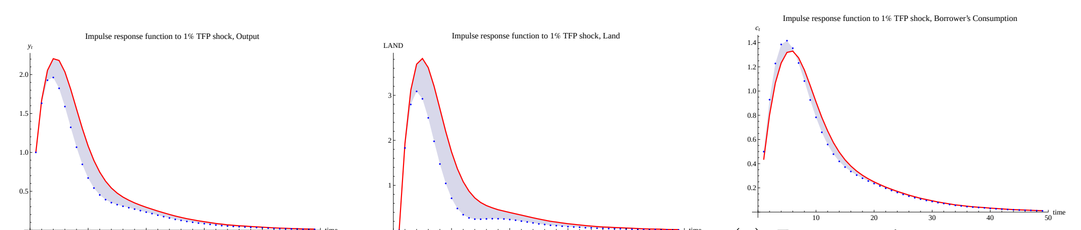
(b) Borrower's land following 1% productivity shock.

Appendices

A Small model

A.1 Additional dynamics

Figure 14: Responses under learning (solid red) and RE (dotted blue) in case of the underestimation of $\mathbf{M_0}$ but with correct beliefs about the persistence, $M_{33,0} = \rho$.



(a) Output following 1% productivity shock.

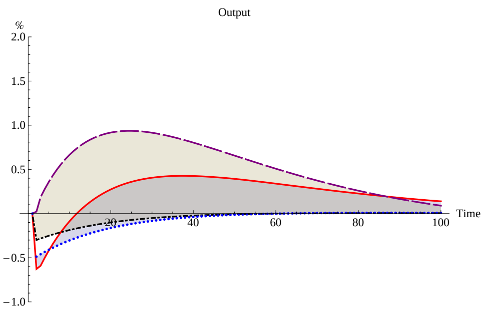
(b) Borrower's land following 1% productivity shock.

(c) Borrower's consumption following 1% productivity shock.

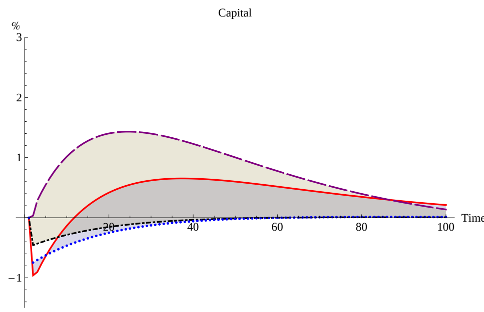
B Collateral constraint model

B.1 Additional dynamics

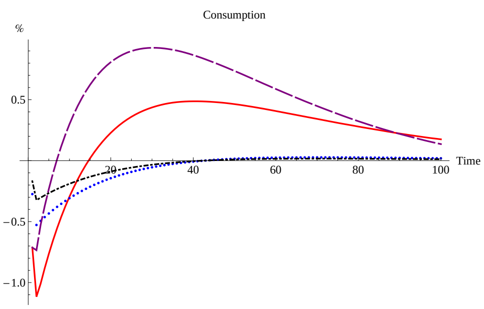
Figure 16: Responses under learning (solid) and RE (dotted) incorrect beliefs regarding the macro-prudential policy with stronger confidence



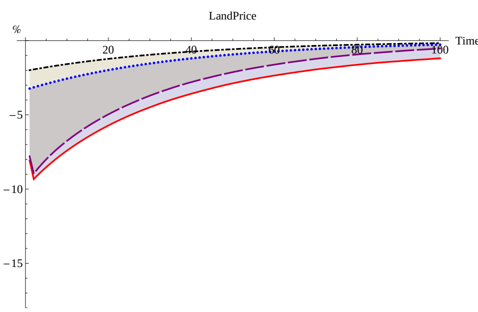
(a) Output following 5% leverage shock.



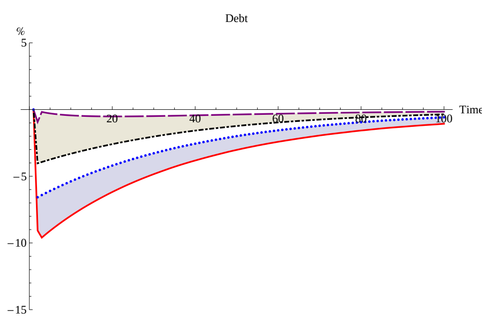
(b) Capital following 5% leverage shock.



(c) Consumption following 5% leverage shock.

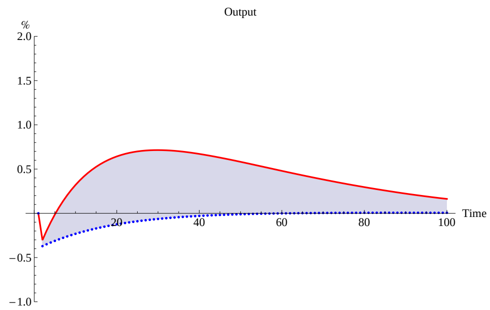


(d) Land price following 5% leverage shock.

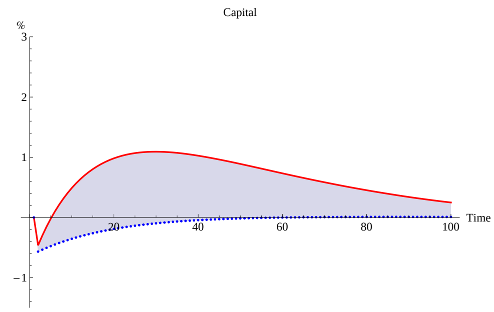


(e) Debt following 5% leverage shock.

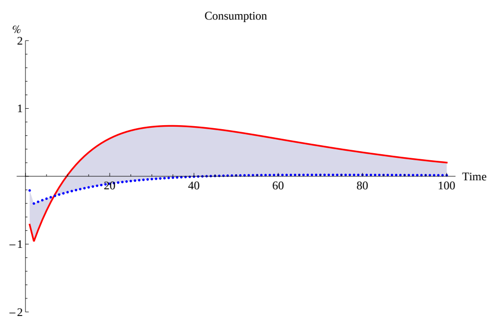
Figure 17: Responses under learning (solid red) and RE (dotted blue) incorrect beliefs regarding the macro-prudential policy with $\varepsilon = -1$ (stochastic debt limit)



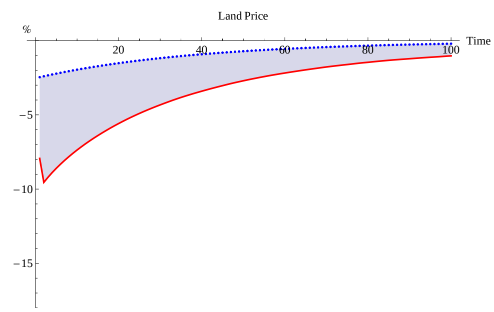
(a) Output following 5% leverage shock.



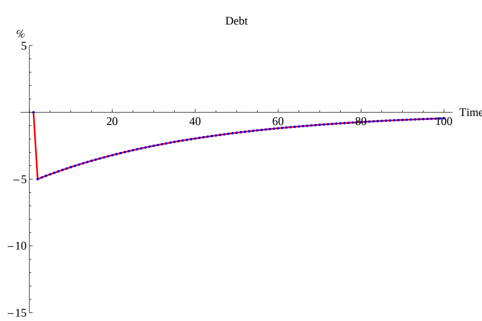
(b) Capital following 5% leverage shock.



(c) Consumption following 5% leverage shock.



(d) Land price following 5% leverage shock.



(e) Debt following 5% leverage shock.