Optimal Monetary Policy at the Zero Lower Bound

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Abstract

We study optimal monetary policy at the zero lower bound. The macroeconomy we study has considerable income inequality which gives rise to a large private sector credit market. Households participating in this market use non-state contingent nominal contracts (NSCNC). A second, small group of households only uses cash and cannot participate in the credit market. The monetary authority supplies currency to cash-using households in a way that changes the price level to provide for optimal risk-sharing in the private credit market and thus to overcome the NSCNC friction. For sufficiently large and persistent negative shocks the zero lower bound on nominal interest rates may threaten to bind. The monetary authority may credibly promise to increase the price level in this situation to maintain a smoothly functioning (complete) credit market. The optimal monetary policy in this model can be broadly viewed as a version of nominal GDP targeting.

Keywords: Zero lower bound, forward guidance, quantitative easing, optimal monetary policy, life cycle economies, heterogeneous households, credit market participation, nominal GDP targeting. *JEL codes*: E4, E5.

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1 Introduction

1.1 The zero lower bound

Following the financial crisis and recession of 2007-2009 in the U.S., the short-term nominal interest rate targeted by monetary policymakers—the policy rate—effectively hit the zero lower bound.[1] In order to provide further policy accommodation subsequent to this event, the Federal Reserve embarked on a two types of policies. One of these is popularly known as "forward guidance"—a promise by the central bank to hold interest rates at the zero lower bound beyond the time when the zero lower bound is actually binding. The other is popularly known as "quantitative easing"—outright purchases of both privately-issued and publicly-issued debt. Both of these types of monetary policy responses have been popular in several other large economies with policy rates constrained by the zero lower bound.

Intense controversy has swirled around these policy responses since their inception. Some widely-cited theoretically-oriented analyses have suggested that forward guidance could provide policy accommodation even when the zero lower bound is a binding constraint on policy.[2] Some of the empirical evidence on the actual effectiveness of forward guidance policies has been mixed.[3] As for quantitative easing, some theoretical analysis suggests that such a policy, at least in its purest form, may have no effect on equilibrium allocations.[4] Yet the empirical evidence on quantitative easing, based in

[1] This paper has benefitted from considerable input on earlier versions, many with a somewhat different focus than the current paper. The authors thank Kevin Sheedy, Patrick Kehoe, and Jonathan Heathcote, as well as the patient comments by seminar participants at the Texas Monetary Conference, Rice University, the Konstanz Seminar on Monetary Theory and Policy (especially our discussant, Keith Kuester), the Swiss National Bank, the European Central Bank, the Bank of Finland, the Minneapolis, St. Louis, Philadelphia, and Chicago Federal Reserve Banks, Deakin University, University of Tasmania, the Meetings of the Society for Economic Dynamics, as well as the Summer Workshop on Money, Banking, Payments and Finance at the Chicago Fed.

[2] See for instance Eggertsson and Woodford (2003).

[3] See for example Kool and Thornton (2012), Filardo and Hofmann (2014), Levin, Lopez-Salido, Nelson, and Yun (2010), and Cole (2015).

[4] Williamson (2012), for instance, suggests that central bank purchases of privately-

part on event study methodology, suggests that in terms of financial market impact these policies seem to have important effects.[5]

In this paper we study optimal monetary policy at the zero lower bound to try to understand whether forward guidance, quantitative easing, or some other policy provides an appropriate response by monetary policymakers when the zero lower bound is encountered. The economy we study includes substantial income inequality which gives rise to a large private credit market. Smooth functioning of this credit market is essential to good macroeconomic performance, but the credit market also contains an important friction in the form of non-state-contingent nominal contracting. In normal times (away from the zero lower bound), monetary policy can mitigate the friction appropriately and thus ensure a smoothly operating (complete) credit market. However, when sufficiently large and persistent negative aggregate shocks hit the economy, the zero lower bound on nominal interest rates may threaten to bind. We study a policy option the monetary authority might consider to maintain a smoothly operating credit market even in this situation. The policy option is a special upward adjustment in the price level.

The price level policy response to an encounter with the zero lower bound identified in this paper is unique compared with responses listed above, and thus may help inform the debate on this topic. In particular, in the framework presented here the forward guidance policy—promising to remain at the zero lower bound beyond the time that the zero lower bound is actually constraining—is not helpful. In addition, while the central bank in this framework could embark on policies that look like quantitative easing—in the sense that private debt could be purchased by the monetary authority—and that such purchases may have real effects, it is not clear how such purchases could be used to maintain credit market completeness in the face of

issued assets have no effects on equilibrium allocations. Similarly, Curdia and Woodford (2010, 2011) develop irrelevance propositions for quantitative easing, and Woodford (2012) suggests that there is no good theoretical basis for the emphasis many central banks have placed on this type of post-ZLB monetary policy.

[5]See for example D'Amico and King (2011), Gagnon, Raskin, Remache, and Sack (2011), Hamilton and Wu (2011), Joyce, Lasaosa, Stevens, and Tong (2010), Krishnamurthy and Vissing-Jorgensen (2011), and Neely (2013).

the NSCNC friction. The monetary policy described in this paper can be broadly understood as a version of nominal GDP targeting.

1.2 What we do

We consider a simple and stylized 241-period general equilibrium life cycle model of quarterly movements in private debt levels, interest rates, and inflation.[6] One-period, privately-issued household debt and currency are the only two assets. We divide the population into two groups, a large number of credit market participants (*a.k.a.,* "credit users") and a small number of credit market non-participants (*a.k.a.,* "cash users"). The credit market has an important friction: Debt contracts must be specified and paid off in nominal terms, and may not be written in state-contingent form. We call this the non-state contingent nominal contract, or NSCNC, friction, and we will discuss it extensively in the main text. There is a stochastic income growth process—an aggregate shock. In particular, aggregate labor productivity growth follows a first-order autoregressive process.[7]

Participant households supply one unit of labor inelastically in each period, but their productivity varies over the life cycle. We study a stylized situation in which participant households' life cycle productivity endowment is exactly peaked in the middle period of the life cycle. The credit-using households will issue debt on net during the first portion of the life cycle and hold positive net assets during the second portion.[8] These households sell their labor productivity units at the prevailing competitive per unit wage, but the wage is stochastic. The real rate of growth in wages is also the real rate of growth of output in this economy.

[6] We think it is important to maintain the quarterly frequency so that the model can be appropriately compared to results from other models. The interest rates we discuss will have a three-month interepretation.

[7] There is no idiosyncratic uncertainty—the only source of uncertainty is the aggregate shock.

[8] While the model is simple and abstract, much of the borrowing that occurs can be thought of as mortgage debt, intended to move the consumption of housing services earlier in the life cycle.

The relatively small group of credit market non-participants, the currency users, are precluded from the credit market altogether. They cannot borrow from or lend to any other household. Their productivity endowment profile is flat and intermittent, so that they can earn income only sporadically (facing the same stochastic wage per productivity unit as the participant households). These agents wish to consume at times when income is unavailable. Thus they are in key respects very different from the households in the credit market participant group.[9] To smooth consumption, the non-participant households use currency issued by the central bank. The price level in the economy will be determined by the currency demand of this cash-using group, subject to the aggregate labor productivity shock. The central bank supplies currency to the economy's cash-using households and can effectively control the price level of the economy through this channel.

Critically, the credit market participants in this model who hold positive net assets—the "savers"—could in principle use either cash or credit. We will ensure that the debt issued by relatively young credit market participants will pay a higher real return and so the savers will prefer to hold this privately-issued debt rather than the publicly-issued currency. This means the net nominal interest rate will be positive. We think of zero nominal interest rates as indicating that the publicly-issued currency is competing directly in real rate of return against the privately-issued paper of relatively young households, distorting their ability to sell their paper at an appropriate price and leading to inefficient outcomes in the credit market. Policy will seek to avoid this situation and therefore keep nominal interest rates away from zero if possible.

Because the credit market is so large relative to the cash-using contingent, we analyze the model as if the optimal monetary policy is one that completes the credit market.[10] We think of the policymaker as having a

[9]This segment of society can be roughly viewed as the underbanked sector. Some estimates suggest that about 8 percent of US households are unbanked, and as many as 20 percent are underbanked (they have a bank account but use alternative financial services). See Burhouse and Osaki (2012).

[10]We think of this large credit market assumption as analogous to the "cashless limit"

hierarchical mandate: (1) Provide for smoothly functioning (*i.e.*, complete) credit markets—one might think of this as "financial stability," and (2) keep inflation relatively low by hitting an exogenously given inflation target (which for convenience we assume to be zero), in order not to harm the cash-using segment of society too much in pursuit of the first goal.[11]

1.3 Main findings

The stationary equilibrium of this economy naturally generates substantial levels of privately-issued household debt relative to GDP. We first show that if credit market participants were allowed to use state-contingent contracts, a stationary equilibrium exists in which the real interest rate in the credit market fluctuates in tandem with the aggregate shock—that is, with the aggregate growth rate of the economy.[12] The price level can be kept constant in this situation. The private credit market transforms the unequal income across participant cohorts alive at a date t into perfectly equal consumption. Each credit market participant would, in effect, have an *equity share* in the income of the credit sector of the economy earned at date t. This is a first-best risk-sharing outcome for the credit sector of this economy under the homothetic preferences we have assumed.

With non-state contingent nominal contracting, credit market participant households will contract nominal amounts of credit with a fixed nominal interest rate one period in advance. We show that in this situation, the central bank can influence the price level each period to provide the otherwise missing state-contingency through a counter-cyclical price level policy. In this circumstance, all cohorts alive at date t will again consume exactly equal amounts, and the real interest rate will again equal the output growth rate each period. Participant households will again have an equity share in

assumption made in the sticky price literature. For a discussion, see Woodford (2003).

[11]The Fed of course has an additional goal related to employment, but labor supply is inelastic in this paper. We intend to analyze this issue in future research. For a model in which labor markets are incomplete (as opposed to credit markets in this paper), see Kocherlakota (2013).

[12]In this sense the credit market sector of the economy is dynamically efficient.

the income of the credit sector of the economy, and this again constitutes optimal risk-sharing for the private credit market. A monetary policy in this class will replicate the complete credit markets allocation from a risk-sharing perspective.[13] We call this the complete credit markets policy.

The policy described above will work well for relatively small shocks—small enough that the net nominal rate of interest always remains positive. However, for certain shock realizations the net nominal interest rate required to implement the complete credit market policy may threaten to encounter the zero lower bound. We discuss a policy option the monetary authority can use in order to maintain complete credit markets. The *price level approach* involves a promise to engineer an increase in the price level one period in the future sufficient to keep the net nominal interest rate positive. This promise is sufficient to ensure that the net nominal interest rate remains positive and the complete credit market policy remains intact.[14] However, this policy has a drawback: The price level policy harms cash-using households relative to the policy away from the zero lower bound. As additional shocks hit the economy, the zero lower bound situation will eventually dissipate and special policy actions will prove temporary.

We conclude that in economies where the key friction is NSCNC and the net nominal interest rate threatens to encounter the zero lower bound, monetary policymakers may wish to respond with a price level increase. A chief rival to this response observed in actual economies—forward guidance on the length of time the economy will remain at the zero lower bound beyond the time when that bound is actually binding—would be inappropriate in the theory presented here. And, while the central bank in this model could purchase privately-issued debt, and such purchases could have real effects, it is unclear how such purchases could meet the complete credit markets objective we have set out for the policymaker in this paper. We will discuss interpretations of the monetary policy in this paper as nominal GDP targeting in

[13] See Sheedy (2014) for a discussion of these issues in related models.

[14] If the zero bound is encountered in subsequent periods, the same policy action has to be repeated.

the main text.

1.4 Recent related literature

Williamson (2012) studies quantitative easing and related issues in a model related to those of Lagos and Wright (2005), Rocheteau and Wright (2005), Sanches and Williamson (2010), and Berentsen, Camera, and Waller (2007). In the section that analyzes the purchase of privately-issued debt, analogous to the mortgage-backed securities purchased by the Federal Reserve in recent years, he concludes that, "At best, central bank purchases of private assets have no effects on prices or quantities in the model."[15] Curdia and Woodford (2010, 2011) reach similar conclusions in extended versions of sticky-price-based New Keynesian models. In fact, these papers contain irrelevance propositions for quantitative easing.

The empirical literature on the effects of quantitative easing includes D'Amico and King (2011), Gagnon, Raskin, Remache, and Sack (2011), Hamilton and Wu (2011), Joyce, Lasaosa, Stevens, and Tong (2010), Krishnamurthy and Vissing-Jorgensen (2011), and Neely (2013). Many of these papers use, at least in part, event study methodology around the dates of significant surprise announcements related to quantitative easing. A broad conclusion is that the observed financial market impacts following the surprise announcements are statistically significant.

The present paper follows in a tradition of monetary theory that emphasizes asset market participation and non-participation. The superior rate of return that can be earned by asset market participant savers then generates a positive nominal interest rate in the economy, and risk sharing can be a key concern of policymakers. Some analysis with this flavor includes Alvarez,

[15] In more recent work, Williamson (2014) does find a role for quantitative easing, but an unconventional one, in a model that relies on limited collateral in credit markets. In similar vien, Araujo, Schommer, and Woodford (2013) consider endogenous collateral constraints in conjunction with private credit markets and ask whether the size and composition of the central bank balance sheet can affect equilibrium outcomes. They find that it can in some circumstances but perhaps not in a way that provides a close or appropriate substitute for ordinary monetary policy that is constrained by the zero lower bound.

Lucas, and Weber (2001) and Zervou (2013).

The monetary features of models related to the one presented in this paper have been studied by Azariadis, Bullard, and Smith (2001) and Bullard and Smith (2003*a*, 2003*b*).[16] These papers feature spatial separation which creates the possibility that privately-issued liabilities like the ones discussed in the present paper circulate in exchange. However, we do not study such possibilities in the present paper. The privately-issued debt is always repaid by the issuer in the following period.

The general equilibrium life cycle model we use has recently been used to analyze issues related to monetary policy and the zero lower bound by Eggertsson and Mehrotra (2014). Their model, like ours, takes advantage of the natural credit market that exists in the life cycle framework, and they use it to study deleveraging, debt dynamics, and issues related to the zero lower bound. They focus on sticky prices as the key friction, whereas we concentrate on NSCNC. In our model, this friction gives a role for monetary policy related to credit market performance. Also, Eggertsson and Mehrotra (2014) follow authors like Benhabib, Schmitt-Grohe, and Uribe (2001), Bullard (2010), and Caballero and Farhi (2015) in modeling the zero lower bound as at least potentially a permanent outcome. In the present paper, the zero lower bound can be encountered because of large and persistent aggregate shocks, but is ultimately temporary.

A paper that is similar in spirit to ours, although different in details, is Buera and Nicolini (2014). They study an economy with an important role for a credit market along with a cash-in-advance friction. In their model, heterogeneous firms borrow against collateral, and large shocks can push the economy to the zero lower bound. Like us, they find that policy trade-offs at the zero lower bound are novel compared to the ones generally emphasized in the literature.

The NSCNC approach takes as an inspiration observed nominal mortgage and related household debt contracts, and accordingly we think it has natural appeal. However, we do not spend time in this paper trying to defend or

[16] See also Gomis-Porqueras and Haro (2009).

subvert the use of this particular friction. We simply assume NSCNC and look at the implications for the conduct of monetary policy. This friction has a long history in discussions of monetary and fiscal policy. Bohn (1988), for instance, presented a theory in which a government may wish to use inflation to change the real value of the debt in response to shocks as a substitute for changing distortionary tax rates. Chari, Christiano, and Kehoe (1991), Chari and Kehoe (1999), Schmitt-Grohe and Uribe (2004), and Siu (2004) debated the extent of inflation volatility required to complete financial markets, coming to differing conclusions in models with and without sticky prices. In the current paper, we have flexible prices but no taxation or fiscal policy of any kind, and the inflation volatility required to complete credit markets is the same as the volatility of the real output growth rate.

Sheedy (2014) provides a comprehensive analysis of an environment where the NSCNC friction plays a key role.[17] Sheedy (2014) also considers a situation in which both sticky price and NSCNC frictions are present, and argues that the NSCNC friction is the more important of the two in a calibrated case. In addition, Sheedy (2014) provides extensive background on the NSCNC friction. Koenig (2013) also provides an analysis of monetary policy in an economy with the NSCNC friction present. The economy there is a two-period case, but the mechanism used to achieve the complete credit markets outcome is the same.

Garriga, Kydland, and Sustek (2013) consider the effect of non-state contingent nominal contracting in housing markets on equilibrium allocations. Their analysis is quantitative-theoretic with a given monetary policy. They find the non-state contingent nominal contracting friction can be quite significant, and suggest that the nature of mortgage contracting has important implications for the impact of monetary policy on the economy. Doepke and Schneider (2006) present empirical evidence that household balance sheets are comprised in large part of nominal liabilities and assets, and find sub-

[17] Bullard (2014) provides comments on the Sheedy paper and suggests that results may generalize to a class of models like the present one. Werning (2014) also comments on Sheedy and discusses the possible effects of idiosyncratic uncertainty. There is no idiosyncratic uncertainty in the present paper.

stantial redistributional effects from unexpected movements in inflation.

In this paper, stationary equilibrium real rates of return are closely related to the real rate of growth in the economy—in fact, we design the model so that the one-period real rate of return in the private credit market is exactly equal to the real output growth rate, which in turn is driven solely by the pace of growth in labor productivity. Versions of this result are a general feature of models in this class, but the exact correspondence between the pace of real growth and the real interest rate in the private credit market is due to the somewhat stylized set of assumptions we use to design the model.

2 Environment

2.1 Segmented markets

Households are divided into two types, labeled "participants" and "non-participants." We also refer to these two types as "credit users" and "cash users," respectively.[18] Both participant and non-participant household cohorts are atomistic, identical, and have mass one, and so we will analyze each participant and each non-participant cohort as if there were only one member. Households live in discrete time for $T+1=241$ periods, which we think of as corresponding to a quarterly model in which households begin economic life with zero assets in their early $20s$ and continue until death. We insist on this time period structure as it allows an interpretation of results in quarterly terms, although only the quarterly interpretation hinges on the particular choice of T. We choose $T+1$ to be an odd number in order to have a convenient and specific peak period for participant productivity endowment profiles. A new cohort of households enters the economy each period such that there is no population growth. The economy continues into the infinite past, so that $-\infty < t < +\infty$. The only assets in the economy are consumption loans in the credit market and currency. Loan contracts

[18] There are no borrowing constraints, and debt is always fully repaid. There is no role for collateral. For alternative theories that emphasize collateral and come to different conclusions, see Williamson (2014) and Araujo, Schommer, and Woodford (2013).

are for one period,[19] are not state-contingent, and are expressed in nominal terms—as we have already discussed, we call this the non-state contingent nominal contracting friction, or NSCNC. Labor is supplied inelastically but households have different levels of labor productivity at different stages of the life cycle. Prices are flexible.

2.2 Stochastic structure

There is an exogenous real wage $w(t)$ which follows

$$w(t+1) = \lambda(t, t+1) w(t), \tag{1}$$

with $w(0) > 0$.[20] We allow the gross rate of real wage growth between any dates t and $t+1$, $\lambda(t, t+1)$, to follow a standard autoregressive process. In particular, $\lambda(t, t+1)$ follows

$$\lambda(t, t+1) = (1-\rho)\lambda + \rho\lambda(t-1, t) + \sigma\eta(t+1), \tag{2}$$

where the unadorned $\lambda > 1$ represents the average gross growth rate, $\rho \in (0, 1)$, $\sigma > 0$, and $\eta(t+1) \sim N(0, 1)$. This stochastic process will work well to make the key points we wish to emphasize.

It will sometimes be useful below to refer explicitly to actual realizations of the stochastic process governing $\lambda(t, t+1)$. We will denote realizations of this process by $\lambda^r(t, t+1)$.

2.3 Timing protocol

A timing protocol determines the role of information in the credit sector of the economy. At any period t, credit-using agents enter with one-period nominal contracts carrying an interest rate $R^n(t-1, t)$ that were based on the

[19] In Sheedy (2014), debt contracts can have long maturities. See also Garriga, Kydland, and Sustek (2013).

[20] This assumption can also be thought of as a aggregate linear production technology in which one productivity unit produces one unit of the good, subject to a multiplicative productivity shock. Then $\lambda(t, t+1)$ is the growth in productivity between dates t and $t+1$.

expected growth rate between period $t-1$ and t, that is, $E_{t-1}\left[\lambda\left(t-1,t\right)\right]$, as well as expected inflation between period $t-1$ and t. Therefore, at the beginning of each date t, households hold nominal contracts which depend on $\lambda\left(t-2,t-1\right)$. Nature moves first at date t and draws a value of $\eta\left(t\right)$ implying a value of $\lambda\left(t-1,t\right)$, the productivity growth rate between date $t-1$ and date t. The monetary policymaker then moves next and chooses a value for its monetary policy instrument. Given these choices, credit-using households make decisions to consume and save via non-state contingent nominal consumption loan contracts for the following period, carrying a nominal interest rate $R^{n}\left(t,t+1\right)$.

2.4 Participant productivity endowments

The productivity endowments of the credit market participant households[21] are given by $e=\left\{e_s\right\}_{s=0}^{T}$. This notation means that each household entering the economy has productivity endowment e_0 in their first period of activity, e_1 in the second, and so on up to e_T. We use

$$e_s=f\left(s\right)=\mu_0+\mu_1 s+\mu_2 s^2+\mu_3 s^3+\mu_4 s^4 \tag{3}$$

such that $f\left(0\right)=0$, $f\left(60\right)=57/100$, $f\left(120\right)=1$, $f\left(180\right)=57/100$, and $f\left(240\right)=0$. Solving these five equations yields the values for μ_i, $i=0,...,4$. This is a stylized endowment profile which emphasizes that near the beginning and end of the life cycle productivity is near or equal to zero. This endowment profile is displayed in Figure 1.

Credit market participant households supply labor inelastically. They sell the productivity units they have at a particular stage in the life cycle in a labor market which pays a competitive real wage $w\left(t\right)$ per efficiency unit. This means that different households will earn considerably different amounts of income (high income inequality at every date), and that total real income in the credit portion of the economy at date t will be given by $w\left(t\right)\sum_{s=0}^{T}e_s$. The bulk of participant income will be earned in the middle

[21] Non-participant households have a different productivity endowment pattern and are discussed below.

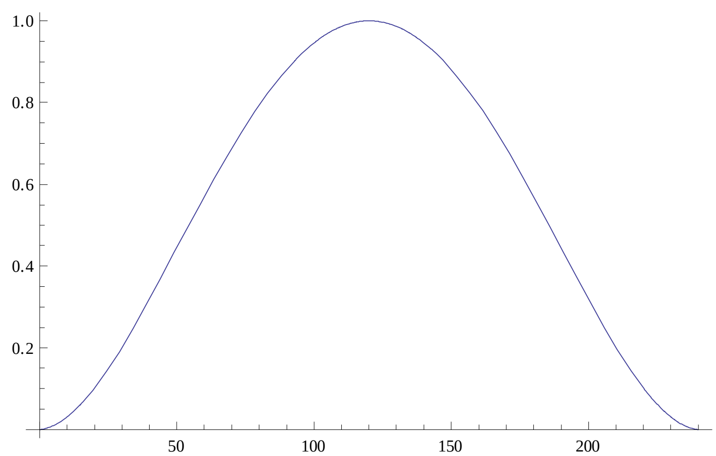

Figure 1: A schematic productivity endowment profile for credit market participant households. The profile is symmetric and peaks in the middle period of the life cycle. Total real income in the credit sector at date t is this profile multiplied by $w(t)$. About 50 percent of the households earn 75 percent of the income in the credit sector.

portion of life. The productivity profile is also symmetric. This means that there will be an exact balance between the need for saving into relative old age and the need for borrowing in relative youth. This in turn means that along a non-stochastic balanced growth path the gross real interest rate will be equal to the average gross growth rate of the economy, $R = \lambda$. This will be an important benchmark for this paper as it will make the discussion particularly simple and transparent.

2.5 The participant household problem

Let $c_i(t)$ denote the real value of consumption of the credit market participant cohort i at date t. The cohort entering the economy at date $i = t$

maximizes expected utility[22]

$$\max_{\{c\}} E_t \sum_{s=0}^{T} \ln c_t\left(t+s\right). \tag{4}$$

In writing the constraints for this maximization we note that the participant households holding positive assets ("savers") will not hold currency because the real rate of return on currency will be lower than the real rate of return on private debt in all states of the world in the stationary equilibria we choose to study. To these credit market participants, currency is an inferior asset—accordingly, we do not include choices of currency holdings in the budget constraints for participant households.

We express all quantities in real terms, except for consumption loans, which, because of the NSCNC friction, are expressed in nominal terms. We will denote net nominal loan amounts of the participant cohort i at date t by $a_i\left(t\right)$, and we interpret negative values as borrowing. We will convert these to real values by dividing by the aggregate price level $P\left(t\right)$ at date t. Since all other variables in the participant households' budget constraints are expressed in real terms, price levels will appear only in tandem with nominal assets a. Given these considerations, the participant household entering the economy at date t faces a sequence of budget constraints that can be expressed as

$$c_t\left(t\right) \leq e_0 w\left(t\right) - \frac{a_t\left(t\right)}{P\left(t\right)}, \tag{5}$$

$$c_t\left(t+1\right) \leq e_1 w\left(t+1\right) + R^n\left(t,t+1\right)\frac{a_t\left(t\right)}{P\left(t+1\right)} - \frac{a_t\left(t+1\right)}{P\left(t+1\right)}, \tag{6}$$

$$\ldots$$

$$c_t\left(t+T\right) \leq e_T w\left(t+T\right) + R^n\left(t+T-1,t+T\right)\frac{a_t\left(t+T-1\right)}{P\left(t+T-1\right)}, \tag{7}$$

[22] This formulation means that the households do not discount the future. In life cycle economies, the discount factor does not have to be less than unity, and so to keep results especially transparent and stark we present results with a discount factor equal to one. A discount factor less than one could easily be incorporated, but results would not be quite as transparent as we have them here.

where $R^n(t,t+1)$ is the one-period gross nominal rate of return on loans originated at date t and maturing at date $t+1$ in the credit sector of the economy.[23] The sequence of budget constraints can be written as a single consolidated budget constraint

$$\begin{aligned}
c_t(t) &+ \frac{P(t+1)}{P(t)}\frac{c_t(t+1)}{R^n(t,t+1)} \\
&+ ... + \frac{P(t+T)}{P(t)}\frac{c_t(t+T)}{R^n(t,t+1)\cdot ... \cdot R^n(t+T-1,t+T)} \\
&\le e_0 w(t) + \frac{P(t+1)}{P(t)}\frac{e_1 w(t+1)}{R^n(t,t+1)} \\
&+ ... + \frac{P(t+T)}{P(t)}\frac{e_T w(t+T)}{R^n(t,t+1)\cdot ... \cdot R^n(t+T-1,t+T)}. \qquad (8)
\end{aligned}$$

This budget constraint is standard. It will be convenient to denote the right hand side of (8) as

$$\begin{aligned}
\Xi_t(t) &= e_0 w(t) + \frac{P(t+1)}{P(t)}\frac{e_1 w(t+1)}{R^n(t,t+1)} \\
&+ ... + \frac{P(t+T)}{P(t)}\frac{e_T w(t+T)}{R^n(t,t+1)\cdot ... \cdot R^n(t+T-1,t+T)}. \qquad (9)
\end{aligned}$$

A benchmark in this paper will be the nonstochastic version of this problem. The no uncertainty case can be thought of as $\sigma = 0$ and $\lambda(-1,0) = \lambda$. The economy grows along a balanced growth path at gross rate λ. For simplicity, let us assume the central bank pursues a constant inflation policy, for example $P(t+1)/P(t) = \pi^* \ \forall t$. Then the gross nominal interest rate $R^n = \lambda\pi^*$ and the choice of first period real consumption for the household entering the economy at date t is given by

$$c_t(t) = \frac{w(t)\sum_{i=0}^{T} e_i}{T+1}. \qquad (10)$$

That is, the participant household entering the economy at date t desires to consume $1/241$ of the right hand side of the budget constraint. In the stationary equilibria we study, this amount will turn out to be $1/241$ of the real

[23] We use the notational convention throughout this paper that R represents gross real returns in the credit market and that other interest rates are differentiated by a superscript.

income available in the credit sector of the economy at date t. Other households alive at date t—those that entered the economy at earlier dates—will solve similar problems, except that they will generally carry non-zero asset holdings into the remainder of their life over which they are optimizing. We will study stationary equilibria for $t \in (-\infty, \infty)$ where these asset holdings are consistent with the stationary equilibrium. In these situations, the participant households that entered the economy earlier than date t will also wish to consume $1/241$ of income available in the credit sector of the economy at date t, and they will adjust their asset holding to accommodate this desire.

2.6 The non-participant household problem

Non-participant households are precluded from using the credit market. Like their participant agent counterparts, they live $T + 1 = 241$ periods. We will discuss these agents according to their stage of life $s = 0, 1, ..., 239, 240$. In stage of life 0, these agents are inactive. They do not consume, nor do they earn labor income. In odd-dated stages of life, these agents have a productivity endowment $\gamma \in (0, 1)$. We will think of this γ value as being fairly low—in addition, there is no life cycle aspect to the value of γ. The households entering the economy at date t then earn income $\gamma w\,(t + s)$, $s > 0$, $s = 1, 3, 5, ..., 239$. In the even-dated stages of life, the non-participant households consume. The period utility for households born at date t in these periods is $\ln c_t\,(t + s)$, $s = 2, 4, 6, ..., 240$. In each odd stage of life, these households solve a two-period problem, discounting all future two period problems they will face to zero.

The non-participant agents evidently earn income only intermittently, as they are endowed with productivity units only in the odd-dated stages of life. They move income into periods when they need to consume, the even-dated periods, by holding currency. With upward sloping wages during their lifetime (that is, the average gross real growth rate $\lambda > 1$), the households will not wish to carry currency beyond one period, because in the next two-period cycle they will have more income. Thus along the balanced growth path there is no reason to save beyond one period, and so these households will simply save all income in the quarters they work by holding currency. Nevertheless, for especially low values of $\lambda(t, t+1)$ these households may possibly wish to hold currency to aid consumption beyond the current even period into the next even period—but, we assume they discount this possibility completely. Accordingly, the cash-using households will solve a series of two period problems, saving all income earned by holding currency, and then consuming everything before working (supplying labor inelastically with productivity γ) again in the following period.[24]

Some non-participants will have labor income at a date in which other non-participants will wish to consume. That is, some will be in an even stage of life $s = 2, 4, ...$ while others will be in an odd stage of life $s = 1, 3, ...$ However, we do not allow credit between these agents. Only currency can change hands between odd-dated and even-dated agents. The even-dated agents wishing to consume will use their cash to buy consumption from the odd-dated agents.

This stylized design of the cash-using segment of the economy will deliver a conventional money demand, buffeted by the aggregate shock to productivity. The price level will be determined in this sector of the economy.

2.7 The fiscal authority

We make assumptions to keep the policy actions of the fiscal authority (*a.k.a.* the government) strictly limited in this economy, so that we can describe the effects of a monetary intervention in isolation. For example, if we model the fiscal authority as one that levies distortionary taxes, provides useful government services, wastes resources, or some combination of these, then the monetary policy effects we wish to describe would be more difficult to interpret as they would depend in part on the particular fiscal arrangements assumed. We do not deny that such considerations are important, but for the purposes of this paper we want to rule out such possibilities and concentrate

[24] This form of the two-period problem eliminates any steady state in which no agent wishes to hold currency.

on what the monetary authority might reasonably be able to accomplish on its own with the fiscal authority sidelined.

Accordingly, we assume the fiscal authority does not tax, nor does it spend on government programs, nor does it waste resources. In fact, the fiscal authority does not interact with any agent other than the central bank. The fiscal authority has a storage technology that it can use to store the consumption good. It is the only agent with access to the storage technology. The real rate of return on the storage is exogenously equal to the gross ex post real rate of return in the credit market, $R(t, t+1)$.[25]

This storage technology assumption is of course not entirely realistic, nor is it meant to be. The storage of the fiscal authority will simply provide a record of the real seigniorage revenues received by the government from the central bank over long periods of time.

2.8 The monetary authority

The monetary authority (*a.k.a.,* the central bank) views the large but incomplete private credit market as the primary focus of monetary policy. Policymakers have a hierarchical mandate, in which (1) The primary goal is to overcome the NSCNC friction in the credit market;[26] and (2) A secondary goal is to hit an exogenously given inflation target on average, here taken to be zero for convenience. The secondary goal ensures that the policymaker does not harm the relatively small, cash-using segment of the society too much in pursuit of the first goal.

The central bank is independent and operates at zero cost. We define independence to mean that the central bank transacts with agents in the

[25] This assumption is convenient, but our results do not hinge on the assumed rate of return on the storage technology.

[26] A poorly functioning credit market could be a key concern for policymakers in this economy. As an extreme case, consider the situation where the credit market breaks down completely, and participant households simply consume according to their income in a particular period. In that case, some households at the beginning and end of the life cycle would be unable to consume at all. The value of a population-weighted social welfare function would tend toward negative infinity.

economy through arm's-length market transactions at competitive prices. The agents on the other side of these transactions include credit market participant households, non-participant households, and the fiscal authority.

The central bank's interaction with other agents in the model takes the following form. In the cash-using, non-participant household sector, the central bank supplies currency by selling it to even-dated households at the competitive market price. The central bank acquires some of the consumption good in this process. This quantity of consumption is then *lent* to the fiscal authority in exchange for debt that promises to repay the loan at the real rate of interest prevailing in the credit market. The fiscal authority uses its storage technology to store the consumption good. In the following period, the fiscal authority repays the loan from the central bank with interest in units of the consumption good. But the central bank then offers the proceeds from the loan repayment plus additional seigniorage earned during that period back to the government as a loan in exchange for new debt issued by the fiscal authority.[27] In this way, the seigniorage revenue earned over a long period of time is simply consumption stored by the fiscal authority, and the central bank holds a growing stock of debt issued by the fiscal authority as an asset. The stock of government debt held by the central bank represents the total past seniorage plus interest delivered to the fiscal authority. This process can continue forever in the stationary equilibria we study, because the central bank never retires any of the currency issued.

This collection of central bank transactions with other agents in the economy creates a real-valued central bank balance sheet. The central bank's balance sheet has total outstanding currency as a liability and accumulated government-issued debt as an asset, similar to the actual Federal Reserve balance sheet during ordinary times. For instance, as of December 31, 2006, the balance sheet of the Federal Reserve System reported about $784 billion in assets held as U.S. government securities. This was about 90 percent of all assets reported. Liabilities included $783 billion in Federal Reserve notes

[27] This statement assumes the economy is not at the zero lower bound, as we discuss below.

outstanding, about 93 percent of all liabilities reported. Total capital was reported as about \$31 billion.[28] This pre-crisis balance sheet is similar to the pre-quantitative easing balance sheet in this model, in which government securities constitute 100 percent of assets, currency outstanding constitutes 100 percent of liabilities, and there is no capital.

The central bank can also trade at market prices with credit market participant households. Each period, when the central bank is repaid by the fiscal authority and is earning additional seniorage revenue, it has access to a relatively large amount of the consumption good. It can sell a portion of its consumption holdings to households in the credit market in exchange for privately-issued debt. This debt will earn the real rate of return prevailing in the credit market. This is like the central bank making direct purchases of "mortgage-backed securities" or other privately-issued debt. We will discuss what such a scheme may or may not accomplish later in the paper.

3 The monetary policy problem

We will describe the monetary policymaker as wishing to complete credit markets by influencing the value of the price level at each date t. As we will show in the next subsection, in this model the policymaker will be able to influence the price level without any control error, so that in effect the policymaker can simply choose the price level at each date. This aspect of the model is of course unrealistic, but the point here is to demonstrate what the optimal monetary policy would look like if such precise control were feasible. Keeping this type of assumption in place is akin to the analysis in the simplest versions of New Keynesian models in which shocks can be offset perfectly by the policymaker through appropriate adjustment of the nominal interest rate.

[28] See the *Annual Report of the Board of Governors of the Federal Reserve System 2007*, p. 359.

3.1 Controlling the price level through currency provision

How is it that the monetary policymaker can control the price level in this model? The policymaker supplies currency, $H(t)$, to the non-participant households—the cash users. The total real value of currency outstanding in the economy at date t is given by $H(t)/P(t)$. We normalize the date 0 currency level to $H(0) = 1$.

A consideration of the problem of non-participant households indicates that there will be $T/2$ cohorts at an odd-dated stage of the life cycle demanding currency at each date t and that these cohorts each have income $\gamma w(t)$. This means the real demand for currency at date t, which we will denote by $h^d(t)$, will be given by

$$h^d(t) = \frac{\gamma T}{2} w(t). \tag{11}$$

The total real value of currency in circulation at date t will have to be held by these households. Equality of supply and demand in the currency market means

$$\frac{H(t)}{P(t)} = \frac{\gamma T}{2} w(t). \tag{12}$$

The central bank chooses the rate of currency creation between any two dates $t-1$ and t, $\theta(t-1,t)$, written as

$$H(t) = \theta(t-1,t) H(t-1). \tag{13}$$

This implies

$$\frac{\gamma T}{2} w(t) P(t) = \theta(t-1,t) \frac{\gamma T}{2} w(t-1) P(t-1) \tag{14}$$

which can be written as

$$\theta(t-1,t) = \frac{P(t)}{P(t-1)} \frac{w(t)}{w(t-1)}. \tag{15}$$

Equation (15) can be read as follows. As the economy is entering date t, the values of $P(t-1)$ and $w(t-1)$ are taken as given. The timing protocol for the economy means that nature moves first and chooses a growth rate $\lambda(t-1,t)$ of real wages, and hence a value for $w(t)$. This means that the central bank, moving after nature, can choose the gross rate of currency creation $\theta(t-1,t)$ to determine a value for $P(t)$.

We conclude that under the assumptions we have outlined, the policymaker can in effect choose the appropriate price level directly in this economy. This choice of $P(t)$ is sufficient to characterize equilibrium in the cash-using sector of the economy.

3.2 Possible policy choices for the rate of currency creation

There are some interesting choices for θ that will turn out not to be optimal in this model, but which provide good benchmarks for comparison. The central bank could, for instance, choose $\theta(t-1,t) = 1 \ \forall t$, in which case a fixed stock of currency would simply trade hands each period between odd-dated and even-dated agents in the currency market. The price level would then fluctuate in response to shocks. We will call this the *fixed currency stock rule*. Another interesting possibility is that the policymaker chooses θ in order to maintain $P(t) = P(t-1) = 1 \ \forall t$ (or any other constant), where we normalize the date 0 price level $P(0) = 1$. We will call this the *price stability rule*. The price stability rule is, broadly speaking, the type of policy advice that would stem from simple New Keynesian models assuming sticky prices. A variant of the price stability rule is that θ is chosen to produce a constant rate of increase in the price level. We will call this an *inflation targeting rule*. Of course, the price stability rule is simply an inflation targeting rule in which the gross inflation target is equal to 1, and the net inflation target is equal to zero. In many simple New Keynesian analyses, the net inflation target is taken to be zero instead of a positive value.

3.3 Nominal interest rates

A critical aspect of the economy we are studying is that the net nominal interest rate has to be positive in order to maintain a dichotomy between the credit sector (in which currency is an option for savers but is never used) and the cash sector (in which credit is not allowed by assumption). Participant households contract by fixing the nominal interest rate one period in advance. From the participant households Euler equation, the non-state contingent nominal interest rate, $R^n(t, t+1)$, is given by

$$R^n(t, t+1)^{-1} = E_t \left[\frac{c_t(t)}{c_t(t+1)} \frac{P(t)}{P(t+1)} \right]. \tag{16}$$

We sometimes call this the contracted nominal interest rate. The E_t operator indicates that households must use information available as of the end of period t before the realization of $\eta(t+1)$.[29] In the equilibria we study, the equity share feature means that all cohorts have the same expectation of the consumption growth rate, so that (16) suffices to determine the contract rate. For example, for agents that entered the economy in any period $t-j$, the nominal contract will specify

$$R^n(t, t+1)^{-1} = E_t \left[\frac{c_{t-j}(t)}{c_{t-j}(t+1)} \frac{P(t)}{P(t+1)} \right], \tag{17}$$

but this expectation will be the same as (16). We will return to this expression to check if and when the zero lower bound threatens to become a binding constraint on monetary policy.

Whether the zero lower bound is encountered depends jointly on the expected behavior of consumption as well as the expected policy of the monetary authority embodied in a policy rule for the price level. For example, given a constant price level policy, the zero lower bound would threaten when the expected net consumption growth rate is less than zero. This, in turn, would occur when a sufficiently large negative shock $\eta(t)$ occurs today, such that, given the serial correlation in the stochastic process, the expected net

[29] For further discussion of this, see Chari and Kehoe (1999).

consumption growth rate one period in the future is negative. Thus, at least for this particular policy rule, the zero lower bound would threaten in cases of large negative disturbances—"recessions."

4 Stationary equilibrium

4.1 General considerations

In this economy, stationary equilibrium can be described as a stochastic sequence $\{R^n(t-1,t), P(t)\}_{t=-\infty}^{+\infty}$ in which households maximize utility subject to the constraints, markets clear, and the monetary policymaker credibly adheres to a given rule which determines $P(t)$. We think of the economy as continuing into the infinite past as well as into the infinite future and describe stationary competitive equilibria. For the credit sector, the problem as we have stated it is one of heterogeneous households facing an aggregate shock. Accordingly, we need to track the distribution of asset holdings among the 241 households in the model in order to calculate the stationary equilibrium. However, under particular monetary policy rules as we describe them, the calculation of this stationary equilibrium will be relatively simple.[30] This is because, conditional on the realization of the shock at date t and the perfectly credible monetary policy rule, the economy is nonstochastic. Key quantities like consumption and asset holdings are linear in the current wage realization $w(t)$.

There will be a given distribution of asset holdings across cohorts at any date t in the economy. We set the date zero distribution of asset holdings to be consistent with the stationary equilibrium under the proposed monetary policy.

The key condition for stationary equilibrium is that total asset holding

[30] See Garriga, Kydland, and Sustek (2013) for calculations of incomplete markets equilibria with the NSCNC friction present. All the stationary equilibria in the current paper will have complete markets because of the optimal monetary policy.

in the credit sector must sum to zero at each date t. This means

$$\frac{A(t)}{P(t)} = \frac{a_{t-T+1}(t) + ... + a_{t-1}(t) + a_t(t)}{P(t)} = 0 \tag{18}$$

where $A(t)$ is aggregate nominal asset holding. Evidently, $P(t)$ is irrelevant in this condition, and so we can simply add up the nominal quantities. This can be written as an expression in expected real wages, nominal interest rates, and price levels along with the given distribution of asset holdings coming into the period.

4.2 The non-stochastic balanced growth path

An important benchmark in this economy is the non-stochastic balanced growth path. Suppose there is no uncertainty, which we can think of as $\sigma = 0$. Coupled with this, assume that the policymaker chooses the price stability rule $P(t) = P(t-1) = 1\ \forall t$ (or any constant value) in order to achieve an exogenously-given net inflation target of zero.

For this special case, first order conditions for the problem defined by (4) and (8) can be solved in terms of $c_t(t)$ and substituted back into (8) to obtain equation (10). We conjecture that the gross real interest rate along the balanced growth path is $R = \lambda$. If we recall the normalization $w(0) = 1$, then $w(t) = \lambda^t w(0) = \lambda^t$, and examination of equation (9) indicates that $\Xi_t(t) = w(t) \sum_{i=0}^{T} e_i$ under the conjecture, and that this quantity is the total real income earned in the credit sector of the economy at date t. This means that the household entering the economy at date t chooses to consume $(1/241)\, w(t) \sum_{i=0}^{T} e_i$. Solutions to the problems of all other households alive at date t indicates that they will also choose to consume this amount. The consumption across the 241 households exhausts total production in the credit sector. This means that the sum of asset holding across these households is zero, and so we conclude that the value $R = \lambda$ establishes a balanced growth path equilibrium of the economy.

Figure 2 shows asset holding by cohort along the non-stochastic balanced growth path. Given that income peaks exactly in the middle of the life cycle,

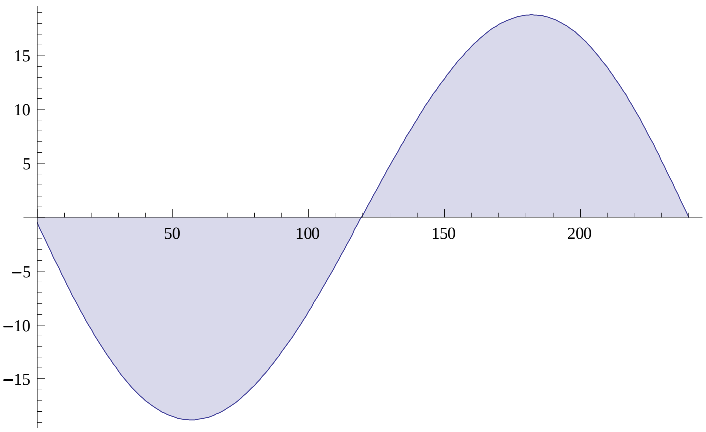

Figure 2: Net asset holding by cohort along the non-stochastic balanced growth path. Borrowing, the negative values to the left, peaks at stage 60 of the life cycle, roughly age 35, while positive assets peak at stage of life 180, roughly age 65. About 25 percent of the population holds about 75 percent of the assets.

participant households will borrow on net in the first half of the life cycle and hold positive net assets in the second half. Net borrowing peaks at stage of life 60 (age 35), and net asset holding peaks at stage of life 180 (age 65). The wealth distribution for participant households is very uneven. If the endowment pattern were perfectly triangle-shaped, then 25 percent of the participant households would hold 75 percent of the assets.

Figure 3 shows the level of household income by cohort and the level of consumption by cohort for this case. In the picture, income by cohort is the bell-shaped curve multiplied by $w(t) = 1$, but the shape is always the same because income is linear in the current real wage. Income is quite uneven across participant cohorts. If the productivity endowment pattern was perfectly triangle-shaped, then 50 percent of households would earn 75 percent of participant sector income at each date. Consumption, on the other hand, is always a completely flat line. Altogether, the ranking for the credit sector of the economy is that the wealth distribution is the most unequal, the income distribution is somewhat less unequal, and the consumption distribution is

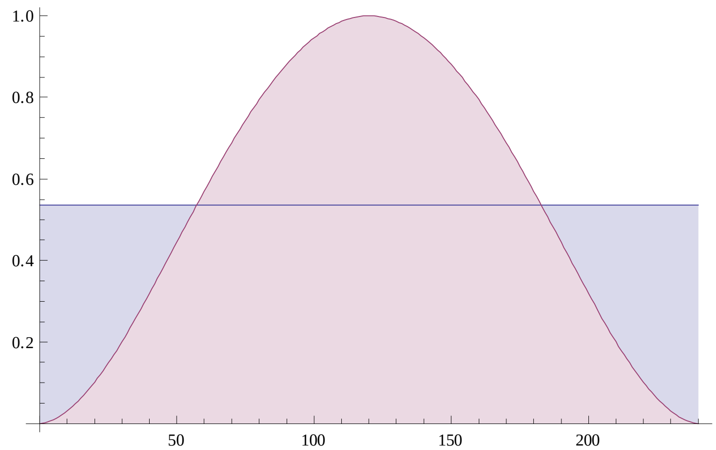

Figure 3: Schematic representation of consumption, the flat line, versus income, the bell shaped curve, by cohort along the non-stochastic balanced growth path with $w(t) = 1$. The private credit market completely solves the point-in-time (cross-sectional) income inequality problem.

perfectly equal.[31]

Importantly, following Sheedy (2014), we can think of the fact that all credit sector cohorts choose to consume $(1/241)\, w(t) \sum_{i=0}^{T} e_i$ as the idea that these households have an "equity share" in the credit sector of the economy—they split up the total available real income at date t as equal real per capita consumption. Equity share contracts are optimal under the homothetic preferences we have assumed. Even though income at date t is very different across households, the private credit market ensures that each household consumes an equal portion of the total real income in the credit sector—the private credit market completely solves the cross-sectional income inequality problem. In the next period, total real income in the credit sector will be higher by a factor λ, but this extra real income will also be split evenly among households alive in the next period. This balanced growth path helps to benchmark the first-best outcome in the credit market.[32]

[31]This only applies to the credit sector. The economy of course has an additional dimension of income inequality because of the cash-using sector.

[32]If we follow a particular participant household from the time they enter the economy until they exit, consumption would increase at gross rate λ each period.

What about the non-participant, cash-using households? Equation (15) indicates that the pace of currency creation $\theta = \lambda$ along the non-stochastic balanced growth path, given the price stability rule. The gross nominal interest rate (16) would also be larger than unity, as $R^n = \lambda > 1$, so the net nominal interest rate would always be positive. This is an important part of the non-stochastic benchmark.

4.3 Complete markets without monetary policy

We turn now to the stochastic case. For the purposes of this sub-section, we eliminate the cash-using sector of the economy as well as the NSCNC friction. This means there is only an economy with credit market participants trading consumption loans, and they have no option to turn to cash—meaning that the zero lower bound is not an issue. Because there is no NSCNC friction, the households can indeed write state-contingent contracts. What would state-contingent contracting look like and how could the stationary equilibrium be characterized? This will offer another benchmark before solving the optimal monetary policy problem.

We will conjecture and then verify a complete markets stationary equilibrium with state-contingent contracting as follows. We conjecture that the gross real interest rate $R(t, t+1)$, $\forall t$, is always equal to the realized gross rate of wage growth between the same dates, $\lambda^r(t, t+1)$, in such a stationary equilibrium. Consideration of equation (9) indicates that, under this conjecture the right hand side of the budget constraint can be written as $w(t) \sum_{i=0}^{T} e_i$, that is, the constraint is linear in $w(t)$. Given the timing protocol of the model, $w(t)$ is known to households at date t when optimization takes place. This means that households solve a non-stochastic problem under the conjecture at date t. The set of non-stochastic problems for the 241 households has a known solution, as shown in the last sub-section, namely that each household consumes $(1/241)\, w(t) \sum_{i=0}^{T} e_i$, an "equity share" in the real output of the economy at date t. In addition, this solution implies $A(t) = 0$ $\forall t$ and this verifies the conjectured stationary equilibrium.

What is the nature of this stationary equilibrium? Aggregate as well as individual consumption changes by larger or smaller amounts each period depending on the value of $w(t)$, but proportionately for all agents alive at that date. Accordingly, asset holding also rises and falls each period for each cohort at each date, but in proportion to the value of $w(t)$ at that date. The entire curve in Figure 2, in other words, is multiplied by the realized value of $w(t)$. Along the non-stochastic balanced growth path, $w(t)$ would always increase by a factor λ. In the complete markets stationary equilibrium with state-contingent contracting, $w(t)$ would generally increase by larger or smaller amounts depending on the outcome of the stochastic process at a particular date. This provides a complete characterization of the asset-holding distribution in the economy at each date.

Versions of this complete markets stationary equilibrium with state-contingent contracting will be the target of the optimal monetary policy described in the remainder of the paper. However, the zero lower bound will become part of the analysis. In addition, without state-contingent contracting, the policymaker will be required to engage in an active price level policy.

5 Complete markets with monetary policy

5.1 A complete markets monetary policy rule

We now return to the full stochastic model with the cash sector included and the NSCNC friction operative. However, for the purposes of this section we will assume that the zero lower bound is never encountered. We can think of this as a situation where σ is positive but arbitrarily small, such that the probability of encountering the zero lower bound is vanishingly small. In the next section, we will allow for larger values of σ, and include encounters with the zero lower bound as part of the equilibrium calculation. We take this intermediate step in order to build intuition before proceeding to the case where the ZLB is a binding constraint.

Intuitively, given the full model and small shocks, we can look for ways in which the policymaker may be able to replicate the equity share contract feature that characterizes the complete markets stationary equilibrium of the previous section.

Indeed, in this situation a price level policy exists that will keep the economy in a version of the complete credit markets stationary equilibrium in this stochastic case with monetary policy. The complete credit markets policy can be described as follows. At each date t, nature chooses a growth rate for labor productivity and hence for wages. The monetary policymaker moves after nature and chooses $P(t)$ in such a way as to restore the complete markets allocation. The complete markets policy rule can be written as

$$\begin{aligned} P(t) &= \frac{E_{t-1}[\lambda(t-1,t)]}{\lambda^r(t-1,t)} P(t-1) \\ &= \frac{(1-\rho)\lambda + \rho\lambda(t-2,t-1)}{(1-\rho)\lambda + \rho\lambda(t-2,t-1) + \sigma\eta(t)} P(t-1). \end{aligned} \tag{19}$$

This monetary policy rule is assumed to be completely credible $\forall t$. This rule delivers the inflation target of zero on average. Because $\eta(t)$ appears in the denominator, the price level rule calls for countercylical price level movements.

We conjecture that a complete markets equilibrium exists even under the incomplete markets contract, provided the policymaker follows the complete markets policy rule (19). Consideration of equation (9) indicates that, under this conjecture and given the complete markets policy rule, the right hand side of the consolidated budget constraint can again be written as $w(t)\sum_{i=0}^{T} e_i$, that is, the constraint is linear in $w(t)$. Given the timing protocol of the model, $w(t)$ is known to households at date t when optimization takes place. This means that households solve a non-stochastic problem under the conjecture at date t. The set of non-stochastic problems for the 241 households has a known solution, as shown in the subsection concerning the nonstochastic balanced growth path. This solution indicates that each household consumes $(1/241)\,w(t)\sum_{i=0}^{T} e_i$, an "equity share" in the real output of the credit sector of the economy at date t. In addition, this solution implies $A(t) = 0\ \forall t$ and $R^n(t,t+1)$ is the rate at which the credit market clears. This verifies the conjectured stationary equilibrium.[33]

[33] This result for the low σ case is similar to Sheedy (2014) and Koenig (2013) in related

Intuitively, the policymaker is providing the missing private sector state-contingency under the NSCNC friction.

The cash-using segment of the economy is affected by the countercyclical price level rule (19). Since prices vary in response to shocks, the real return to currency holding, $R^m(t)$, also varies. On average, however, the net inflation rate is zero, the same as it would be under the price stability rule.

The policy (19) suggests counter-cyclical movements in the price level. We can think of the nonstochastic price level trend as the price stability policy $P(t) = 1 \; \forall t$, or a net inflation target of zero. Relative to this trend, the price level will sometimes be above and sometimes be below. In this sense, the price level is below normal when output is growing relatively rapidly, and the price level is above normal when output is growing relatively slowly. In terms of inflation rates, inflation would be relatively high at times when output is growing slowly and inflation would be relatively low when output is growing rapidly. On average, the net inflation rate would be zero (which we have defined as the inflation target here), and the policymaker would achieve the targeted rate of inflation in an average sense. It is the nature of the reaction to shocks which distinguishes the complete markets policy rule from the price stability rule, not the average rate of inflation.

5.2 Interpretation as nominal GDP targeting

Another way to view the optimal monetary policy in the low volatility economy is as nominal income targeting.[34] Nominal GDP, denoted $Y^n(t)$, in this model is the real wage at date t multiplied by the sum of productivity endowments, times the price level at that date:

$$Y^n(t) = P(t)\, w(t) \left[\frac{T\gamma}{2} + \sum_{i=0}^{T} e_i \right]. \tag{20}$$

economies.

[34] For an extensive discussion of interpretations of monetary policiies in this class as nominal income targeting, see Sheedy (2014).

The value of this variable along the non-stochastic balanced growth path is, assuming the normalizations $P(0) = w(0) = 1$,

$$Y^{n,\star}(t) = \lambda^t \left[\frac{T\gamma}{2} + \sum_{i=0}^{T} e_i \right], \tag{21}$$

and in particular, the target nominal GDP at date $t+1$ can be written as

$$Y^{n,\star}(t+1) = \lambda P(t) w(t) \left[\frac{T\gamma}{2} + \sum_{i=0}^{T} e_i \right]. \tag{22}$$

Consider (20) at date $t+1$:

$$Y^n(t+1) = P(t+1) w(t+1) \left[\frac{T\gamma}{2} + \sum_{i=0}^{T} e_i \right] \tag{23}$$

$$= \frac{(1-\rho)\lambda + \rho\lambda(t-1,t)}{(1-\rho)\lambda + \rho\lambda(t-1,t) + \sigma\eta(t+1)} P(t) \lambda^r(t,t+1) w(t) \left[\frac{T\gamma}{2} + \sum_{i=0}^{T} e_i \right] \tag{24}$$

$$= \left[(1-\rho)\lambda + \rho\lambda(t-1,t)\right] P(t) w(t) \left[\frac{T\gamma}{2} + \sum_{i=0}^{T} e_i \right]. \tag{25}$$

Comparison of (22) and (25) indicates that the monetary policy would return nominal GDP exactly to the target nominal GDP path each period provided $\rho = 0$, that is, in the case of no serial correlation. When shocks are serially correlated, the policy returns nominal GDP partially toward target depending on the value of ρ.[35]

5.3 Fiscal implications

The fiscal implications of the optimal policy (19) are as follows. The monetary authority sells new currency to non-participant households at each date

[35] We note that this model is unlikely to fit macroeconomic data from recent decades, since the monetary policy supporting the stationary equilibrium here has not been the one in use in the largest economies in recent years. Central banks around the world have mostly adopted policies emphasizing stable prices. The historically-observed price stability policy is inappropriate in the economy studied in this paper.

t. The new currency is exchanged for the consumption good, producing real seniorage revenue in terms of the consumption good. The central bank does not consume. Accordingly, it lends the consumption amount to the fiscal authority in exchange for government-issued paper. The government-issued paper promises to pay the same gross ex post real rate of return as in the private credit market, $R(t, t+1)$. The fiscal authority then puts the consumption amount into its storage technology, which also pays a gross ex post real return of $R(t, t+1)$. In the following period, $t+1$, the fiscal authority repays the central bank with the consumption good plus interest, $R(t, t+1)\,x(t)$. But the central bank does not consume, so it again lends this amount plus new seniorage earned at date $t+1$ to the fiscal authority in exchange for newly-issued government paper. Via this process, the amount of the consumption good in the fiscal authority's storage technology rises over time and is equal to

$$x(t+1) = R(t, t+1)\,x(t) + \frac{H(t+1) - H(t)}{P(t+1)}. \tag{26}$$

The central bank's balance sheet has outstanding currency as a liability, and government-issued paper as an asset. The real assets on the central bank balance sheet rise over time and are a reflection of the entire sequence of past seniorage earned through the currency creation process.

5.4 The zero lower bound

When would the economy threaten to encounter the zero lower bound? A consideration of the nominal interest rate in equation (16) indicates that, given the policy rule (19), the net nominal interest rate will be negative if expected net consumption growth is negative. Given the serial correlation in the shock process ρ, such an event could occur for sufficiently negative draws for $\eta(t)$. Moreover, for sufficiently negative shocks the nominal interest rate may be expected to be negative further into the future.

Is there an alternative way to complete credit markets with a monetary policy intervention when the zero lower bound threatens to bind? This is the topic of the next section.

6 Encountering the zero lower bound

6.1 Disruption in the credit market

We now look at the nature of the optimal monetary policy in an economy with normal volatility—that is $\sigma >> 0$. The key characteristic of this case versus the previous section is that the zero lower bound on nominal interest rates may be encountered for certain negative shocks to the economy.

When the zero lower bound is encountered in this economy in expected terms, it involves a serious disruption to monetary arrangements. In the stationary equilibria we have described, the credit market participant households that are past the midpoint of their lifecycle are holding positive assets. As we have it, all of these assets are private paper issued by relatively young households.[36] In our model, this privately-issued paper pays a superior real rate of return and so is preferred by households saving for the latter portion of their life cycle. However, if the zero lower bound is encountered, these saving households will no longer wish to hold the privately-issued paper of the younger agents. Instead, they will want to hold currency issued by the government—entailing a significant shift in money demand. All else equal, this would tend to put upward pressure on the real interest rate in the credit market and downward pressure on the price level. A complete model of this process is beyond the scope of this paper, but the shift of a large segment of the economy's households into money holding would involve a significant transition.

Instead of allowing this type of outcome, in this section we ask what type of additional policy measures might be taken to preserve the complete markets outcome in the credit market during the period when the zero lower bound impinges on monetary policy.

[36] If we tried to size the amount of household debt of this sort in the U.S. economy, we might refer to Mian and Sufi (2011), who suggest an order of magnitude of more than one U.S. annual GDP, or about \$20 to \$25 trillion in today's dollars. We think of this as a large amount of asset holding by relatively older participant households that could become a demand for currency.

6.2 Large negative shocks

When a relatively large negative shock $\eta(t)$ is drawn by nature in this economy, consumption in the current period will fall. This, by itself, is not a concern for the equilibria we have described. However, if there is sufficient serial correlation future consumption growth may also be expected to be negative. The zero lower bound is encountered when, given the price rule in equation (19), expected net consumption growth is negative, as can be seen from equation (16) which pins down the nominal interest rate. This suggests there may be two approaches to avoiding the zero lower bound and thus maintaining the equilibrium allocation that replicates complete credit markets: Either get the price level to increase or get the consumption growth rate to increase. With either approach, the nominal interest rate could potentially remain away from zero, and the credit market will continue to function smoothly with non-state contingent nominal contracts. We will focus primarily on the price level approach.

6.3 Policy when the ZLB threatens

6.3.1 Price level approach

In this section, the central bank announces that if a large negative shock hits the economy at any date t such that the agents would otherwise expect nominal interest rate $R^n(t,t+1) < 1$, the central bank will react by credibly promising to create a higher than usual price level at date $t+1$ such that the zero lower bound condition on the net nominal interest rate does not bind. In this policy scenario, the policy rule (in place for all time) can be described as

$$P(t+1) = \begin{cases} \frac{E_t[\lambda(t,t+1)]}{\lambda^r(t,t+1)} P(t) & \text{if} \quad E_t[\lambda(t,t+1)] > 1, \\ \frac{E_t[\lambda(t,t+1)][1+\vartheta_p(t+1)]}{\lambda^r(t,t+1)} P(t) & \text{if} \quad E_t[\lambda(t,t+1)] \leq 1, \end{cases} \tag{27}$$

where $\vartheta_p(t+1) > 0$ is such that $E_t[\lambda(t,t+1)]\,\vartheta_p(t+1) = 1^+$, and 1^+ represents a value just larger than unity. The top branch of (27) is just the complete markets monetary policy rule (19) of the previous section. Therefore,

(27) can be understood as a generalized version of the policy rule proposed there. The generalization is simply the value of $\vartheta_p(t+1)$.

We conjecture that a complete markets allocation exists even under the incomplete markets contract, provided the policymaker follows the complete markets policy rule (27). Consideration of equation (9) indicates that, under this conjecture and given the complete markets policy rule, the right hand side of the consolidated budget constraint can again be written as $w(t)\sum_{i=0}^{T} e_i$, that is, the constraint is linear in $w(t)$. Given the timing protocol of the model, $w(t)$ is known to households at date t when optimization takes place. This means that households solve a non-stochastic problem under the conjecture at date t. The set of non-stochastic problems for the 241 households has a known solution, as shown in the subsection concerning the nonstochastic balanced growth path. This solution indicates that each household consumes $(1/241)\,w(t)\sum_{i=0}^{T} e_i$, an "equity share" in the real output of the credit sector of the economy at date t. In addition, this solution implies $A(t) = 0\ \forall t$ and this verifies the conjectured stationary equilibrium.

While the policy rule (27) maintains complete markets allocations for participant households, it does have a drawback in that cash-holding households are harmed by the increase in the price level in periods of negative expected consumption growth. These households hold cash to transfer income from periods when it is earned into periods when it is consumed. When the price level is higher than normal, the savers in this group will be forced to consume less than they otherwise would have, and so their utility would be lower. The monetary authority has an inflation target of zero, which is designed to prevent too much damage to the cash-using households in pursuit of completing credit markets. The policy rule (27) backs off of this objective, temporarily allowing higher inflation in periods when the zero lower bound threatens.

Over a long period of time with some sufficiently large shocks with sufficient serial correlation, the equilibrium under the policy rule (27) would create an average inflation rate somewhat higher than the inflation target. However, this effect could be mitigated or eliminated by adopting an inflation target somewhat below zero in times of high expected consumption growth which would then be offset by somewhat higher inflation in times when the zero lower bound threatens to bind. In this sense, the price level policy could maintain an exogenously given inflation target in the very long run, while still dealing effectively with the zero lower bound should it threaten to become binding.

What are the limits on such a policy? A limit to the effectiveness of this policy would be reached if the central bank tried to create such a large increase in the price level that the cash-using segment of the economy simply quits holding currency altogether. As we have described it in Section 2.6 above, the cash-using households would never make such a decision because it would mean they would be unable to consume at all. However, in less extreme formulations such a possibility could easily arise.

6.3.2 A central bank balance sheet approach?

Are there ways to achieve the same complete credit markets outcome when the zero lower bound threatens that look like quantitative easing? As we have it, the monetary authority could credibly promise to buy a sufficient quantity of debt from private sector participant households directly in period $t+1$ in exchange for consumption goods—this would loosely correspond to quantitative easing. This purchase of privately-issued debt will put additional consumption in the hands of the participant households in period $t+1$; potentially this additional consumption could mean that the expected consumption growth rate is higher than it would otherwise be and therefore that the net nominal interest rate will be positive. However, it is unclear how the central bank could use such an intervention to maintain complete credit markets.

Any promise of future consumption faces several issues. First, the consumption injection would have to be viewed by households as permanent income in order to alter behavior. This would mean the central bank would have to buy paper but not require redemption—a fiscal transfer. Second, all households would have to see higher permanent income in proportion to the wage in order to maintain the symmetry of the model. Third and perhaps most importantly, any credible promise of future consumption would affect borrowing and saving in the current period, undoing the desired effect of increasing the expected consumption growth rate. We conclude that it remains unclear whether or how the central bank could use outright purchases of privately-issued debt to engineer a complete markets allocation when the zero lower bound threatens, despite the central bank having real resources at its disposal to do so. Any such policy, if it could be implemented by the monetary authority, would be quasi-fiscal in nature.

7 Conclusions

In this paper we have constructed a stylized model to address core issues in current monetary policy, which, because of the financial crisis of 2007-2009, has become more focused on private credit market behavior. The model includes substantial income inequality, which gives rise to a large and active credit market with some realistic features, including relatively young households wishing to pull consumption forward in the life cycle, relatively old households saving for the later stages of life, and cash-using households that do not participate in the credit market. The net nominal interest rate is positive at all times, which keeps credit market households from wishing to hold cash. A relatively large and persistent negative aggregate shock—that is, a big recession—can cause the zero lower bound on the nominal interest rate to threaten to bind. We have made assumptions that make the analysis particularly simple and tractable, despite the relatively substantial heterogeneity of households and the existence of an aggregate shock to the pace of growth.

The key friction in the model is non-state contingent nominal contracting (NSCNC) in the credit sector. The non-state contingency means that credit market equilibrium will feature inefficient risk sharing if there is no intervention.[37] In this model, the implications of this inefficiency could be substan-

[37] That is, households will not be able to borrow and lend in a way that allows them to smooth consumption over their life cycles appropriately.

tial.[38] However, the fact that the contracting is in nominal terms means that the monetary authority may be able to replace the missing state-contingency with appropriate price level movements, and thus restore efficient risk sharing. This is in fact what happens in the stationary equilibria we study, and this constitutes optimal monetary policy provided the policymaker is focused first on the performance of the relatively large credit market, and only secondarily on maintaining an exogenously-given inflation target on average. The required price level movements are counter-cyclical—meaning that relatively high inflation would be associated with low growth, and relatively low inflation would be associated with high growth, in such a way that the long run average rate of inflation would be unchanged from what it would be under ordinary inflation targeting.

When a big recession occurs under such a policy, the zero lower bound on the nominal interest rate may threaten to bind. What is the policymaker to do in this circumstance, if the objective is to maintain smoothly operating credit markets?

We showed that the monetary authority can still maintain complete markets in this circumstance. This intervention can be implemented via a special price level increase. This keeps the nominal interest rate positive and maintains the complete market allocations for credit market participant households. Nevertheless, this policy also has a drawback: The price level increase harms cash-using households relative to policy away from the zero lower bound.[39]

We think these results may help to inform the debate on monetary policy at the zero lower bound. The suggested price level policy response found here differs from the forward guidance and quantitative easing possibilities listed at the beginning of the paper. In particular, the often-cited policy advice

[38] See Section 2.8 above.

[39] It may be possible for the policymaker to name a short-run inflation target somewhat lower than a long-run inflation target. The policymaker would know that the zero lower bound would sometimes be encountered with a certain probability, and would plan to respond to that situation with a price level increase. In total, in the very long run, the long-run inflation target would be maintained. This would in some sense do the least average amount of damage to cash-using households.

that the monetary authority should promise to remain at the zero lower bound beyond the time that the bound is actually a binding constraint is unhelpful in the present model. Quantitative easing—central bank purchases of privately-issued debt—can be implemented within this setting and may have real effects, but it is unclear how or whether such purchases could be used to maintain complete credit market allocations. Since the policy implications appear to be quite different, a fruitful area for future research may be to try to better understand whether sticky prices or NSCNC is the more relevant friction for policymakers in this situation.[40]

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